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Statistical Convergence in 2-Normed Spaces

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Abstract. In this paper we study statistical convergence in 2-normed spaces. We show that some properties of statistical convergence of real number sequences also hold for sequences in 2-normed spaces. We also define the notion of a statistical Cauchy sequence in 2-normed spaces. We obtain a criteria for a sequence in a 2-normed spaces to be a statistical Cauchy sequence.

Keywords: Natural density; Statistical convergence; Statistical cauchy sequence; 2-normed spaces.

1. Introduction and Background

The concept of statistical convergence was introduced by Steinhaus [12] in 1951 (see also [4]) and also independently by Buck [1] for real and complex sequences. Statistical convergence has been discussed in number theory [3], trigonometric series [13], and summability theory [5]. This notion of convergence has been characterized using measure theory [10]. Also, Maddox [8] extended the concept for sequences in any Hausdorff locally convex topological vector spaces. In the case of real sequences, Fridy [6] obtained the statistical analogue of the Cauchy criterion for convergence. In [9], these concepts were applied to Turnpike theory.

The concept of 2-normed spaces was initially introduced by Gähler in [7]. Since then, many researchers have studied this concept and obtained various results, see for instance [2, 11].

We extend the work of Gähler to define statistical convergence in 2-normed spaces. We also define a notion of statistical Cauchy sequence in 2-normed spaces and obtain a criteria for sequence in 2-normed spaces to be statistical Cauchy sequence in 2-normed spaces.

Briefly, we recall some facts connecting with statistical convergence. Statistical convergence is a generalization of the usual notion of convergence that parallels the usual theory of convergence. If K is subset of positive integers $\mathbb{N}$, then K_n denotes the set $\{k \in K : k \leq n\}$. The natural density of K is given by $\delta(K) = \lim_{n\to\infty} \frac{|K_n|}{n}$, where $|K_n|$ denotes the number of elements in K_n, provided this limit exists. Clearly, finite subsets have natural density zero and $\delta(K^c) = 1 - \delta(K)$ where $K^c = \mathbb{N} \backslash K$, i.e., the complement of K. If $K_1 \subseteq K_2$ and K_1 and K_2 have natural densities then $\delta(K_1) \leq \delta(K_2)$. Moreover, if $\delta(K_1) = \delta(K_2) = 1$, then $\delta(K_1 \cap K_2) = 1$ (see [5]).

A real numbers sequence $x = \{x_n\}$ is statistically convergent to L provided that for every $\varepsilon > 0$ the set $\{n \in \mathbb{N} : |x_n - L| \geq \varepsilon\}$ has natural density zero; in this case we write $st - \lim_n x_n = L$. The sequence $x = \{x_n\}$ is statistically Cauchy sequence if for each $\varepsilon > 0$ there is positive integer $N = N(\varepsilon)$ such that $\delta\left(\left\{n \in \mathbb{N} : \left|x_n - x_{N(\varepsilon)}\right| \geq \varepsilon\right\}\right) = 0$ (see [6]).

If $x = \{x_n\}$ is a sequence that satisfies some property P for all n except a set of natural density zero, then we say that $\{x_n\}$ satisfies P for "almost all n" and we abbreviate "a.a. n".

The following theorem will help us in establishing our results.

Theorem 1.1. [6] *The following statements are equivalent:*

(i) *x is statistically convergent sequence;*

(ii) *x is statistically Cauchy sequence;*

(iii) *x is sequence for which there is a convergent sequence y such that $x_n = y_n$ for a.a. n.*

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A 2-norm on X is a function $\|.,.\| : X \times X \to \mathbb{R}$ which satisfies

(i) $\|x, y\| = 0$ if and only if x and y are linearly dependent;

(ii) $\|x, y\| = \|y, x\|$;

(iii) $\|\alpha x, y\| = |\alpha| \|x, y\|$, $\alpha \in \mathbb{R}$;

(iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$.

The pair $(X, \|.,.\|)$ is then called a 2-normed space [7].

As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with the 2-norm $\|x, y\| :=$ the area of the parallelogram spanned by the vectors x and y, which may be given explicitly by the formula

$$\|x, y\| = |x_1 y_2 - x_2 y_1|, \quad x = (x_1, x_2), \quad y = (y_1, y_2).$$

Let $(X, \|.,.\|)$ be a finite dimensional 2-normed space and $u = \{u_1, ..., u_d\}$ be a basis of X. We can define the norm $\|.\|_\infty$ on X by

$$\|x\|_\infty := \max\{\|x, u_i\| : i = 1, ..., d\}.$$

Associated to the derived norm $\|.\|_{\infty}$, we can define the (closed) balls $B_u(x,\varepsilon)$ centered at x having radius ε by

$$B_u(x,\varepsilon) := \{y : \|x-y\|_{\infty} \leq \varepsilon\},$$

where $\|x-y\|_{\infty} := \max\{\|x-y,u_j\|,\ j=1,...,d.\}$.

Now, we introduce the notion of statistical convergence in 2-normed spaces and give the main results of the paper.

2. Statistical Convergence of 2-Normed Spaces

Definition 2.1. *Let $\{x_n\}$ be a sequence in 2-normed space $(X, \|.,.\|)$. The sequence $\{x_n\}$ is said to be statistically convergent to L, if for every $\varepsilon > 0$, the set*

$$\{n \in \mathbb{N} : \|x_n - L, z\| \geq \varepsilon\}$$

has natural density zero for each nonzero z in X, in other words $\{x_n\}$ statistically converges to L in 2-normed space $(X, \|.,.\|)$ if

$$\lim_{n\to\infty} \frac{1}{n} |\{n : \|x_n - L, z\| \geq \varepsilon\}| = 0$$

for each nonzero z in X. It means that for every $z \in X$,

$$\|x_n - L, z\| < \varepsilon \quad a.a.\ n.$$

In this case we write $st-\lim_{n\to\infty} \|x_n, z\| := \|L, z\|$.

Remark 2.2. If $\{x_n\}$ is any sequence in X and L is any element of X, then the set

$$\{n \in \mathbb{N} : \|x_n - L, z\| \geq \varepsilon, \text{ for every } z \in X\} = \emptyset,$$

since if $z = \overrightarrow{0}$ (0 vector), $\|x_n - L, z\| = 0 \not\geq \varepsilon$ so the above set is empty.

If the sequence in 2-normed space $(X, \|.,.\|)$ is convergent then it is also statistically convergent since the natural density of any finite set is zero. The converse of this claim does not hold in general, which can be seen by the following examples.

Example 2.3. Let $X = \mathbb{R}^2$ be equipped with the 2-norm by the formula

$$\|x,y\| = |x_1y_2 - x_2y_1|, \ x = (x_1,x_2), \ \ y = (y_1,y_2).$$

Define the $\{x_n\}$ in 2-normed space $(X, \|.,.\|)$ by

$$x_n = \begin{cases} (1,n) & \text{if} \quad n = k^2, k \in \mathbb{N}, \\ \left(1, \dfrac{n-1}{n}\right) & \text{otherwise.} \end{cases}$$

and let $L=(1,1)$ and $z=(z_1,z_2)$. If $z_1=0$ then

$$K=\{n\in\mathbb{N}:\|x_n-L,z\|\geq\varepsilon\}=\emptyset$$

for each z in X. Hence $\delta(K)=0$. Therefore we have $z_1\neq 0$. For each $\varepsilon>0$ and $z\in X$, $\left\{n\in\mathbb{N}:n\neq k^2,\,k\leq\frac{|z_1|}{\varepsilon}\right\}$ is a finite set, so

$$\{n\in\mathbb{N}:\|x_n-L,z\|\geq\varepsilon\}=\left\{n\in\mathbb{N}:n=k^2,\,k\geq\sqrt{\frac{\varepsilon}{|z_1|}+1}\right\}\cup\{\text{finite set}\}.$$

Therefore,

$$\frac{1}{n}\left|\{n\in\mathbb{N}:\|x_n-L,z\|\geq\varepsilon\}\right|=\frac{1}{n}\left|\left\{n\in\mathbb{N}:n=k^2,\,k\geq\sqrt{\frac{\varepsilon}{|z_1|}+1}\right\}\right|\cup\frac{1}{n}0(1)$$

for each z in X. Hence, $\delta(\{n\in\mathbb{N}:\|x_n-L,z\|\geq\varepsilon\})=0$ for every $\varepsilon>0$ and $z\in X$, which means that $st-\lim_{n\to\infty}\|x_n,z\|=\|L,z\|$. But, the sequence $\{x_n\}$ is not convergent to L.

Example 2.4. Define the $\{x_n\}$ in 2-normed space $(X,\|.,.\|)$ by

$$x_n=\begin{cases}(0,n) & \text{if} \quad n=k^2,k\in\mathbb{N},\\ (0,0) & \text{otherwise.}\end{cases}$$

and let $L=(0,0)$ and $z=(z_1,z_2)$. Then for every $\varepsilon>0$ and $z\in X$

$$\{n\in\mathbb{N}:\|x_n-L,z\|\geq\varepsilon\}\subset\left\{1,4,9,16,...,n^2,...\right\}.$$

We have that $\delta(\{n\in\mathbb{N}:\|x_n-L,z\|\geq\varepsilon\})=0$, for every $\varepsilon>0$ and $z\in X$. This implies that $st-\lim_{n\to\infty}\|x_n,z\|=\|L,z\|$. But, the sequence $\{x_n\}$ is not convergent to L.

A sequence which converges statistically need not be bounded. This fact can be seen from Example 2.3 and Example 2.4.

The uniqueness of the limit of a statistically convergent sequence can be verified as follows.

Theorem 2.5. *Let $\{x_n\}$ be a sequence in 2-normed space $(X,\|.,.\|)$ and $L,L'\in X$. If $st-\lim_{n\to\infty}\|x_n,z\|=\|L,z\|$ and $st-\lim_{n\to\infty}\|x_n,z\|=\|L',z\|$, then $L=L'$.*

Proof. Assume $L\neq L'$. Then $L-L'\neq\overrightarrow{0}$, so there exists a $z\in X$, such that $L-L'$ and z are linearly independent (such a z exists since $d\geq 2$). Therefore

$$\|L-L',z\|=2\varepsilon,\ \text{ with } \varepsilon>0.$$

Now

$$\begin{aligned}2\varepsilon &= \|(L-x_n)+(x_n-L'),z\|\\ &\leq \|x_n-L',z\|+\|x_n-L,z\|.\end{aligned}$$

So $\{n : \|x_n - L', z\| < \varepsilon\} \subseteq \{n : \|x_n - L, z\| \geq \varepsilon\}$. But $\delta(\{n : \|x_n - L', z\| < \varepsilon\}) = 0$ then contradicting the fact that $x_n \to L'\,(stat)$. ■

The next theorem is an analogue of a result of Fridy [6].

Theorem 2.6. *Let a sequences* $\{x_n\}$ *and* $\{y_n\}$ *in* 2- *normed space* $(X, \|.,.\|)$. *If* $\{y_n\}$ *is a convergent sequence such that* $x_n = y_n$ *a.a.* n *then* $\{x_n\}$ *is statistically convergent.*

Proof. Suppose $\delta(\{n \in: \mathbb{N} : x_n \neq y_n\}) = 0$ and $\lim_{n\to\infty} \|y_n, z\| = \|L, z\|$. Then, for every $\varepsilon > 0$ and $z \in X$

$$\{n \in \mathbb{N} : \|x_n - L, z\| \geq \varepsilon\} \subseteq \{n \in \mathbb{N} : \|y_n - L, z\| \geq \varepsilon\} \cup \{n \in \mathbb{N} : x_n \neq y_n\}.$$

Therefore

$$\begin{aligned} &\delta(\{n \in \mathbb{N} : \|x_n - L, z\| \geq \varepsilon\}) \\ &\leq \delta(\{n \in \mathbb{N} : \|y_n - L, z\| \geq \varepsilon\}) + \delta(\{n \in \mathbb{N} : x_n \neq y_n\}). \end{aligned} \tag{1}$$

Since $\lim_{n\to\infty} \|y_n, z\| = \|L, z\|$ for every $z \in X$, the set $\{n \in \mathbb{N} : \|y_n - L, z\| \geq \varepsilon\}$ contain finite number of integers. Hence, $\delta(\{n \in \mathbb{N} : \|y_n - L, z\| \geq \varepsilon\}) = 0$. Using inequality (1), we get

$$\delta(\{n \in \mathbb{N} : \|x_n - L, z\| \geq \varepsilon\}) = 0$$

for every $\varepsilon > 0$ and $z \in X$. Consequently, $st - \lim_{n\to\infty} \|x_n, z\| = \|L, z\|$. ■

We next provide a proof of the fact that statistical limit operation for sequence in 2-normed space $(X, \|.,.\|)$ is linear with respect to summation and scalar multiplication.

Theorem 2.7. *Let a sequences* $\{x_n\}$ *and* $\{y_n\}$ *in* 2*-normed space* $(X, \|.,.\|)$ *and* $L, L' \in X$ *and* $a \in \mathbb{R}$. *If* $st - \lim_{n\to\infty} \|x_n, z\| = \|L, z\|$ *and* $st - \lim_{n\to\infty} \|y_n, z\| = \|L', z\|$, *for every nonzero* $z \in X$, *then*

(i) $st - \lim_{n\to\infty} \|x_n + y_n, z\| = \|L + L', z\|$, *for each nonzero* $z \in X$ *and*

(ii) $st - \lim_{n\to\infty} \|ax_n, z\| = \|aL, z\|$, *for each nonzero* $z \in X$.

Proof. (i) Assume that $st - \lim_{n\to\infty} \|x_n, z\| = \|L, z\|$ and $st - \lim_{n\to\infty} \|y_n, z\| = \|L', z\|$ for every nonzero $z \in X$. Then $\delta(K_1) = 0$ and $\delta(K_2) = 0$ where

$$K_1 = K_1(\varepsilon) := \left\{n \in \mathbb{N} : \|x_n - L, z\| \geq \frac{\varepsilon}{2}\right\}$$

and

$$K_2 = K_2(\varepsilon) := \left\{n \in \mathbb{N} : \|y_n - L', z\| \geq \frac{\varepsilon}{2}\right\}$$

for every $\varepsilon > 0$ and $z \in X$. Let

$$K = K(\varepsilon) := \{n \in \mathbb{N} : \|x_n + y_n - (L + L'), z\| \geq \varepsilon\}.$$

To prove that $\delta(K) = 0$, it suffices to show that $K \subset K_1 \cup K_2$. Suppose $n_0 \in K$. Then

$$\|x_{n_0} + y_{n_0} - (L + L'), z\| \geq \varepsilon. \tag{2}$$

Suppose to the contrary, that $n_0 \notin K_1 \cup K_2$. Then $n_0 \notin K_1$ and $n_0 \notin K_2$. If $n_0 \notin K_1$ and $n_0 \notin K_2$ then $\|x_{n_0} - L, z\| < \frac{\varepsilon}{2}$ and $\|y_{n_0} - L', z\| < \frac{\varepsilon}{2}$. Then, we get

$$\begin{aligned} \|x_{n_0} + y_{n_0} - (L + L'), z\| &\leq \|x_{n_0} - L, z\| + \|y_{n_0} - L', z\| \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} \\ &= \varepsilon \end{aligned}$$

which contradicts (2). Hence $n_0 \in K_1 \cup K_2$, that is, $K \subset K_1 \cup K_2$.

(ii) Let $st - \lim_{n\to\infty} \|x_n, z\| = \|L, z\|$, $a \in \mathbb{R}$ and $a \neq 0$. Then

$$\delta\left(\left\{n \in \mathbb{N} : \|x_n - L, z\| \geq \frac{\varepsilon}{|a|}\right\}\right) = 0.$$

Then, we have

$$\begin{aligned} \{n \in \mathbb{N} : \|ax_n - aL, z\| \geq \varepsilon\} &= \{n \in \mathbb{N} : |a| \|x_n - L, z\| \geq \varepsilon\} \\ &= \left\{n \in \mathbb{N} : \|x_n - L, z\| \geq \frac{\varepsilon}{|a|}\right\}. \end{aligned}$$

Hence, the right hand side of above equality equals 0. Hence, $st - \lim_{n\to\infty} \|ax_n, z\| = \|aL, z\|$ for every nonzero $z \in X$. ■

We introduce the statistical analog of the Cauchy convergence criterion in 2-normed space $(X, \|.,.\|)$.

Definition 2.8. *A sequence $\{x_n\}$ in 2-normed space $(X, \|.,.\|)$ is said to be statistically Cauchy sequence in X if for every $\varepsilon > 0$ and every nonzero $z \in X$ there exists a number $N = N(\varepsilon, z)$ such that*

$$\delta\left(\left\{n \in \mathbb{N} : \left\|x_n - x_{N(\varepsilon,z)}, z\right\| \geq \varepsilon\right\}\right) = 0,$$

i.e., for every nonzero $z \in X$,

$$\left\|x_n - x_{N(\varepsilon,z)}, z\right\| < \varepsilon \quad a.a.\ n.$$

From Theorem 1 of Fridy [6] we have

Theorem 2.9. *Let $\{x_n\}_{n\geq 1}$ be a statistically Cauchy sequence in a finite dimensional 2-normed space $(X, \|.,.\|)$. Then, there exists a convergent sequence $\{y_n\}_{n\geq 1}$ in $(X, \|.,.\|)$ such that $x_n = y_n$ for a.a. n.*

Proof. First note that $\{x_n\}_{n\geq 1}$ is a statistically Cauchy sequence in $(X, \|.\|_\infty)$. Choose a natural number $N(1)$ such that the closed ball $B_u^1 = B_u\left(x_{N(1)}, 1\right)$ contains x_n for a.a. n. Then, choose a natural number $N(2)$ such that the closed ball $B_2 = B_u\left(x_{N(1)}, \frac{1}{2}\right)$ contains x_n for a.a. n. Note that, $B_u^2 = B_u^1 \cap B_2$ also contains x_n for a.a. n. Thus by continuing of this process, we can obtain a sequence $\{B_u^m\}_{m\geq 1}$ of nested closed balls such that $diam\,(B_u^m) \leq \frac{1}{2^m}$. Therefore, $\cap_{m=1}^{\infty} B_u^m = \{A\}$. Since each B_u^m contains x_n for a.a. n, we can choose a sequence of strictly increasing natural numbers $\{T_m\}_{m\geq 1}$ such that

$$\frac{1}{n}\left|\{n \in \mathbb{N} : x_n \notin B_u^m\}\right| < \frac{1}{m}, \text{ if } n > T_m.$$

Put $W_m = \{n \in \mathbb{N} : n > T_m,\ x_n \notin B_u^m\}$ for all $m \geq 1$, and $W = \cup_{m=1}^{\infty} W_m$. Now, define the sequence $\{y_n\}_{n\geq 1}$ as following:

$$y_n = \begin{cases} A & \text{if} \quad n \in W, \\ x_n & \text{otherwise.} \end{cases}$$

Note that, $\lim_{n\to\infty} y_n = A$. In fact, for each $\varepsilon > 0$ choose a natural number m such that $\varepsilon > \frac{1}{m} > 0$. Then for each $n > T_m$, or $y_n = A$, or $y_n = x_n \in B_u^m$, and so in each case, $\|y_n - A\|_\infty \leq diam\,(B_u^m) \leq \frac{1}{2^{m-1}}$. Since $\{n \in \mathbb{N} : y_n \neq x_n\} \subseteq \{n \in \mathbb{N} : x_n \notin B_u^m\}$, we have

$$\frac{1}{n}\left|\{n \in \mathbb{N} : y_n \neq x_n\}\right| \leq \frac{1}{n}\left|\{n \in \mathbb{N} : x_n \notin B_u^m\}\right| < \frac{1}{m}.$$

Hence, $\delta\left(\{n \in \mathbb{N} : y_n \neq x_n\}\right) = 0$. Thus in the space $(X, \|.\|_\infty)$, $x_n = y_n$ for a.a. n. Suppose that $\{u_1, ..., u_d\}$ is a basis for $(X, \|.,.\|)$. Since $\lim_{n\to\infty} \|y_n - A\|_\infty = 0$ and $\|y_n - A, u_i\| \leq \|y_n - A\|_\infty$ for all $1 \leq i \leq d$, $\lim_{n\to\infty} \|y_n - A, z\|_\infty = 0$ for every $z \in X$. It completes the proof. ■

In order to prove the equivalence of Definitions 2.1 and 2.8 we shall find it helpful to use Theorems 2.6 and 2.9.

Theorem 2.10. *Let $\{x_n\}$ be a sequence in 2-normed space $(X, \|.,.\|)$. The sequence $\{x_n\}$ is statistically convergent if and only if $\{x_n\}$ is a statistically Cauchy sequence.*

Proof. Assume that $st-\lim \|x_n, z\| = \|L, z\|$ for every nonzero $z \in X$ and $\varepsilon > 0$. Then, for every $z \in X$,

$$\|x_n - L, z\| < \frac{\varepsilon}{2} \quad \text{a.a. } n,$$

and if $N := N(\varepsilon, z)$ is chosen so that $\left\|x_{N(\varepsilon,z)} - L, z\right\| < \frac{\varepsilon}{2}$, then, we have

$$\begin{aligned} \left\|x_n - x_{N(\varepsilon,z)}, z\right\| &\leq \|x_n - L, z\| + \left\|L - x_{N(\varepsilon,z)}, z\right\| \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} \quad \text{a.a. } n \\ &= \varepsilon \quad \text{a.a. } n. \end{aligned}$$

Hence, $\{x_n\}$ is statistically Cauchy sequence.

Conversely, assume that $\{x_n\}$ is a statistically Cauchy sequence. By Theorem 2.9, there exists a convergent sequence $\{y_n\}$ in $(X, \|.,.\|)$ such that $x_n = y_n$ for a.a. n. By Theorem 2.6, we have $st-\lim \|x_n, z\| = \|L, z\|$ for each z in X. ■

As an immediate consequence of Theorem 6 we have the following result.

Theorem 2.11. *If $\{x_n\}$ is a sequence in 2-normed space $(X, \|.,.\|)$ such that $st\text{-}\lim \|x_n, z\| = \|L, z\|$ for every nonzero $z \in X$, then $\{x_n\}$ has a subsequence $\{x_{n_i}\}$ such that $\lim_{i\to\infty} \|x_{n_i}, z\| = \|L, z\|$ for every nonzero $z \in X$.*

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