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On Real Valued I-Convergent A-Summable Sequence Spaces Defined by Sequences of Orlicz Functions

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Abstract. In this article we introduce some new sequence spaces using I-convergence and sequences of Orlicz functions, and study some basic topological and algebraic properties of these spaces. Also we investigate the relations between these spaces.

Keywords: Ideal; I-convergent; Orlicz function; Matrix transformation.

1. Introduction

The notion of I-convergence was initially introduced by Kostyrko, Salat and Wilczynski [5]. Later on, it was further investigated from the sequence space point of view and linked with the summability theory by Salat, Tripathy and Ziman [13, 14], Tripathy and Hazarika [18, 19, 20] and Kumar and Kumar [7], Subramanian [15], Rath and Tripathy [11], Altin, Et and Tripathy [1], Tripathy and Mahanta [22], Et, et.al [3], Tripathy [16], Tripathy and Sen [24], and many others authors.

Let X be a non-empty set, then a family of sets $I \subset 2^X$ (the class of all subsets of X) is called an *ideal* if and only if for each $A, B \in I$, we have $A \cup B \in I$ and for each $A \in I$ and each $B \subset A$, we have $B \in I$. A non-empty family of sets $F \subset 2^X$ is a *filter* on X if and only if $\phi \notin F$, for each $A, B \in F$, we have $A \cap B \in F$ and each $A \in F$ and each $A \subset B$, we have $B \in F$. An ideal I is called *non-trivial ideal* if $I \neq \phi$ and $X \notin I$. Clearly $I \subset 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on X. A non-trivial ideal $I \subset 2^X$ is called *admissible* if and only if $\{\{x\} : x \in X\} \subset I$. A non-trivial ideal I is *maximal* if there cannot exists any non-trivial ideal $J \neq I$ containing I as a subset. Further details on ideals of 2^X can be found in [5, 23, 21, 25, 17].

Lemma 1.1. [5, Lemma 5.1] *If $I \subset 2^N$ is a maximal ideal, then for each $A \subset N$ we have either $A \in I$ or $N - A \in I$.*

Example 1.2. If we take $I = I_f = \{A \subseteq N : A \text{ is a finite subset}\}$. Then I_f is a non-trivial admissible ideal of N and the corresponding convergence coincide with the usual convergence.

Example 1.3. If we take $I = I_\delta = \{A \subseteq N : \delta(A) = 0\}$ where $\delta(A)$ denote the asymptotic density of the set A. Then I_δ is a non-trivial admissible ideal of N and the corresponding convergence coincide with the statistical convergence.

Recall in [6] that an *Orlicz function* M is continuous, convex, nondecreasing function define for $x > 0$ such that $M(0) = 0$ and $M(x) > 0$. If the convexity of Orlicz function is replaced by sub-additivity that is $M(x + y) \leq M(x) + M(y)$ then this function is called the *modulus function* and characterized Nakano [9] and followed by Ruckle [12]. An Orlicz function M is said to satisfy Δ_2-*condition* for all values of u, if there exists $K > 0$ such that $M(2u) \leq KM(u)$, $u \geq 0$.

Lemma 1.4. *Let M be an Orlicz function which satisfies Δ_2-condition and let $0 < \delta < 1$. Then for each $t \geq \delta$, we have $M(t) < K\delta^{-1}tM(2)$ for some constant $K > 0$.*

Two Orlicz functions M_1 and M_2 are said to be *equivalent* if there exist positive constants α, β and x_0 such that

$$M_1(\alpha) \leq M_2(x) \leq M_1(\beta)$$

for all x with $0 \leq x < x_0$.

Lindenstrauss and Tzafriri [8] studied some Orlicz type sequence spaces defined as follows:

$$\ell_M = \left\{ (x_k) \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}.$$

The space ℓ_M with the norm

$$||x|| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}$$

becomes a Banach space which is called an *Orlicz sequence space*. The space ℓ_M is closely related to the space ℓ_p which is an Orlicz sequence space with $M(t) = |t|^p$, for $1 \leq p < \infty$.

In the later stage, different classes of Orlicz sequence spaces were introduced and studied by Parashar and Choudhury [10], Esi and Et [2] and many others.

Throughout the article N and R denote the set of positive integers and set of real numbers respectively. The zero sequence is denoted by θ.

Let $A = (a_{ki})$ be an infinite matrix of complex numbers. We write $Ax = (A_k(x))$ if $A_k(x) = \sum_i a_{ik}x_k$ converges for each i.

A sequence space E_F is said to be *solid (or normal)* if $(y_k) \in E_F$ whenever $(x_k) \in E_F$ and $|y_k| \leq |x_k|$ for all $k \in N$.

Lemma 1.5. [4, p. 53] *A sequence space E_F is normal implies E_F is monotone.*

2. Some New Sequence Spaces

The following well-known inequality will be used throughout the article. Let $p = (p_k)$ be any sequence of positive real numbers with $0 \leq p_k \leq \sup_k p_k = G$, $D = \max\{1, 2^{G-1}\}$ then

$$|a_k + b_k|^{p_k} \leq D\left(|a_k|^{p_k} + |b_k|^{p_k}\right)$$

for all $k \in N$ and $a_k, b_k \in C$. Also $|a_k|^{p_k} \leq \max\left\{1, |a|^G\right\}$ for all $a \in C$.

The main aim of this article is to introduce the following sequence spaces and examine some topological and algebraic properties of the resulting sequence spaces. Let I be an admissible ideal of the non-empty set S and let $p = (p_k)$ be a bounded sequence of positive real numbers for all $k \in N$. Let $\mathbf{M} = (M_k)$ be a sequence of Orlicz functions and $A = (a_{ki})$ be an infinite matrix and $x = (x_k)$ be a sequence of real or complex numbers. For some $\rho > 0$, we define the following sequence spaces:

$$W^I(\mathbf{M}, A, p) = \left\{ (x_k) \in w : \left\{ n \in N : \frac{1}{n} \sum_{k=1}^{n} \left[M_k \left(\frac{|A_k(x) - L|}{\rho} \right) \right]^{p_k} \geq \varepsilon \right\} \in I, \right.$$

$$W_0^I(\mathbf{M}, A, p) = \left\{ (x_k) \in w : \left\{ n \in N : \frac{1}{n} \sum_{k=1}^{n} \left[M_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} \geq \varepsilon \right\} \in I, \right.$$

$$W_\infty^F(\mathbf{M}, A, p) = \left\{ (x_k) \in w : \sup \frac{1}{n} \sum_{k=1}^{n} \left[M_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} < \infty, \right.$$

and

$$W_\infty^I(\mathbf{M}, A, p) = \Big\{(x_k) \in w : \exists K > 0 \text{ such that}$$
$$\left\{n \in N : \frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(x)|}{\rho}\right)\right]^{p_k} \geq K\right\} \in I.$$

3. Main Results

In this section we examine the basic topological and algebraic properties of these spaces and obtain the inclusion relation between these spaces.

Theorem 3.1. *$W^I(\mathbf{M}, A, p)$, $W_0^I(\mathbf{M}, A, p)$ and $W_\infty^I(\mathbf{M}, A, p)$ are linear spaces.*

Proof. We establish the result for the space $W_0^I(\mathbf{M}, A, p)$ only and for the other cases, it can be proved in a similar way. Let $x = (x_k)$ and $y = (y_k)$ be two elements in $W_0^I(\mathbf{M}, A, p)$. Then there exist $\rho_1 > 0$ and $\rho_2 > 0$ such that

$$A_{\frac{\varepsilon}{2}} = \left\{n \in N : \frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(x)|}{\rho_1}\right)\right]^{p_k} \geq \frac{\varepsilon}{2}\right\} \in I$$

and

$$B_{\frac{\varepsilon}{2}} = \left\{n \in N : \frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(y)|}{\rho_2}\right)\right]^{p_k} \geq \frac{\varepsilon}{2}\right\} \in I.$$

Let α, β be two scalars. By the continuity of each Orlicz function M_k's in the sequence $\mathbf{M} = (M_k)$ the following inequality holds:

$$\begin{aligned}
&\frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(\alpha x + \beta y)|}{|\alpha|\rho_1 + |\beta|\rho_2}\right)\right]^{p_k} \\
\leq& D\frac{1}{n}\sum_{k=1}^{n}\left[\frac{|\alpha|}{|\alpha|\rho_1 + |\beta|\rho_2}M_k\left(\frac{|A_k(x)|}{\rho_1}\right)\right]^{p_k} \\
&+ D\frac{1}{n}\sum_{k=1}^{n}\left[\frac{|\beta|}{|\alpha|\rho_1 + |\beta|\rho_2}M_k\left(\frac{|A_k(y)|}{\rho_2}\right)\right]^{p_k} \\
\leq& DK\frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(x)|}{\rho_1}\right)\right]^{p_k} + DK\frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(y)|}{\rho_2}\right)\right]^{p_k},
\end{aligned}$$

where $K = \max\left\{1, \frac{|\alpha|}{|\alpha|\rho_1 + |\beta|\rho_2}, \frac{|\beta|}{\alpha|\rho_1 + |\beta|\rho_2}\right\}$.

From the above relation we obtain the following:

$$\left\{n \in N : \frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(\alpha x + \beta y)|}{|\alpha|\rho_1 + |\beta|\rho_2}\right)\right]^{p_k} \geq \varepsilon\right\}$$
$$\subseteq \left\{n \in N : DK\frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(x)|}{\rho_1}\right)\right]^{p_k} \geq \frac{\varepsilon}{2}\right\}$$
$$\bigcup\left\{n \in N : DK\frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(\frac{|A_k(y)|}{\rho_2}\right)\right]^{p_k} \geq \frac{\varepsilon}{2}\right\} \in I.$$

This completes the proof. ■

Remark 3.2. It is easy to verify that the space $W_\infty^F(\mathbf{M}, A, p)$ is a linear space.

Theorem 3.3. *The space $W_\infty^F(\mathbf{M}, A, p)$ is a paranormed space (not totally paranormed) with the paranorm g defined by*

$$g(x) = \inf\left\{\rho^{\frac{p_k}{H}} : \sup_k M_k\left(\frac{|A_k(x)|}{\rho}\right) \leq 1, \textit{ for some } \rho > 0\right\},$$

where $H = \max\{1, \sup_k p_k\}$.

Proof. Clearly $g(-x) = g(x)$ and $g(\theta) = 0$. Let $x = (x_k)$ and $y = (y_k)$ be two elements in $W_\infty^F(\mathbf{M}, A, p)$. Then for $\rho > 0$ we put

$$A_1 = \left\{\rho_1 > 0 : \sup_k M_k\left(\frac{|A_k(x)|}{\rho_1}\right) \leq 1\right\}$$

and

$$A_2 = \left\{\rho_2 > 0 : \sup_k M_k\left(\frac{|A_k(y)|}{\rho_2}\right) \leq 1\right\}.$$

Let $\rho_1 \in A_1$ and $\rho_2 \in A_2$. If $\rho = \rho_1 + \rho_2$ then we obtain the following

$$M_k\left(\frac{|A_k(x+y)|)}{\rho}\right) \leq \frac{\rho_1}{\rho_1 + \rho_2}M_k\left(\frac{|A_k(x)|)}{\rho_1}\right) + \frac{\rho_2}{\rho_1 + \rho_2}M_k\left(\frac{|A_k(y)|}{\rho_2}\right).$$

Thus we have

$$\sup_k\left[M_k\left(\frac{|A_k(x+y)|}{\rho}\right)\right]^{p_k} \leq 1$$

and

$$\begin{aligned} g(x+y) &= \inf\left\{(\rho_1 + \rho_2)^{\frac{p_k}{H}} : \rho_1 \in A_1, \rho_2 \in A_2\right\} \\ &= g(x) + g(y) \\ &\leq \inf\left\{\rho_1^{\frac{p_k}{H}} : \rho_1 \in A_1\right\} + \inf\left\{\rho_2^{\frac{p_k}{H}} : \rho_2 \in A_2\right\} \end{aligned}$$

Let $t^m \to L$ where $t^m, L \in C$ and let $g(x^m - x) \to 0$ as $m \to \infty$. To prove that $g(t^m x^m - Lx) \to 0$ as $m \to \infty$. We put

$$A_3 = \left\{ \rho_m > 0 : \sup_k \left[M_k \left(\frac{|A_k(x^m)|}{\rho_m} \right) \right]^{p_k} \leq 1 \right\}$$

and

$$A_4 = \left\{ \rho_l > 0 : \sup_k \left[M_k \left(\frac{|A_k(x^m - x)|}{\rho_s} \right) \right]^{p_k} \leq 1 \right\}.$$

By the continuity of the sequence $\mathbf{M} = (M_k)$ we observe that

$$\begin{aligned} & M_k \left(\frac{|A_k(t^m x^m - Lx)|}{|t^m - L|\,\rho_m + |L|\,\rho_s} \right) \\ \leq\; & M_k \left(\frac{|A_k(t^m x^m - Lx^m)|}{|t^m - L|\,\rho_m + |L|\,\rho_s} \right) + M_k \left(\frac{|A_k(Lx^m - Lx)|}{|t^m - L|\,\rho_m + |L|\,\rho_s} \right) \\ \leq\; & \frac{|t_k^m - L|\rho_m}{|t^m - L|\,\rho_m + |L|\,\rho_s} M_k \left(\frac{|A_k(x^m)|}{\rho_m} \right) \\ & + \frac{|L|\rho_s}{|t^m - L|\,\rho_m + |L|\,\rho_s} M_k \left(\frac{|A_k(x^m - x)|}{\rho_s} \right). \end{aligned}$$

From the above inequality it follows that

$$\sup_k \left[M_k \left(\frac{|A_k(t^m x^m - Lx)|}{|t^m - L|\,\rho_m + |L|\,\rho_s} \right) \right]^{p_k} \leq 1$$

and consequently

$$\begin{aligned} & g(t^m x^m - Lx) \\ =& \inf \left\{ (|t^m - L|\rho_m + |L|\rho_s)^{\frac{p_k}{H}} : \rho_m \in A_3, \rho_s \in A_4 \right\} \\ \leq& |t^m - L|^{\frac{p_k}{H}} \inf \left\{ (\rho_m)^{\frac{p_k}{H}} : \rho_m \in A_3 \right\} + |L|^{\frac{p_k}{H}} \inf \left\{ (\rho_s)^{\frac{p_k}{H}} : \rho_s \in A_4 \right\} \\ \leq& \max \left\{ |t^m - L|, |t^m - L|^{\frac{p_k}{H}} \right\} g(x^m) + \max \left\{ |L|, |L|^{\frac{p_k}{H}} \right\} g(x^m - x). \qquad (1) \end{aligned}$$

Note that $g(x^m) \leq g(x) + g(x^m - x)$ for all $m \in N$. Hence by our assumption the right hand side of the relation (1) tends to 0 as $m \to \infty$ and the result follows. This completes the proof. ■

Theorem 3.4. *Let* $\mathbf{M} = (M_k)$ *and* $\mathbf{S} = (S_k)$ *be sequences of Orlicz functions. Then the following hold:*

(i) $W_0^I(\mathbf{S}, A, p) \subseteq W_0^I(\mathbf{M} \circ \mathbf{S}, A, p)$, *provided* $p = (p_k)$ *be such that* $G_0 = \inf p_k > 0$.

(ii) $W_0^I(\mathbf{M}, A, p) \cap W_0^I(\mathbf{S}, A, p) \subseteq W_0^I(\mathbf{M} + \mathbf{S}, A, p)$.

Proof. (i) Let $\varepsilon > 0$ be given. Choose $\varepsilon_1 > 0$ such that $\max\left\{\varepsilon_1^G, \varepsilon_1^{G_0}\right\} < \varepsilon$. Choose $0 < \delta < 1$ such that $0 < t < \delta$ implies that $M_k(t) < \varepsilon_1$ for each $k \in N$. Let $x = (x_k)$ be any element in $W_0^I(\mathbf{S}, A, p)$. Put

$$A_\delta = \left\{ n \in N : \frac{1}{n} \sum_{k=1}^{n} \left[S_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} \geq \delta^G \right\}.$$

Then by the definition of ideal we have $A_\delta \in I$. If $n \notin A_\delta$ we have

$$\begin{aligned} & \frac{1}{n} \sum_{k=1}^{n} \left[S_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} < \delta^G \\ \Rightarrow & \sum_{k=1}^{n} \left[S_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} < n\delta^G \\ \Rightarrow & \left[S_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} < \delta^G, \text{ for } k = 1, 2, 3, ..., n \\ \Rightarrow & S_k \left(\frac{|A_k(x)|}{\rho} \right) < \delta, \end{aligned} \tag{2}$$

Using the continuity of the sequence $\mathbf{M} = (M_k)$ from the relation (2) we have

$$M_k \left(S_k \left(\frac{|A_k(x)|}{\rho} \right) \right) < \varepsilon_1, \text{ for } k = 1, 2, 3, ..., n.$$

Consequently we get

$$\begin{aligned} & \sum_{k=1}^{n} \left[M_k \left(S_k \left(\frac{|A_k(x)|}{\rho} \right) \right) \right]^{p_k} < n. \max\left\{\varepsilon_1^G, \varepsilon_1^{G_0}\right\} < n\varepsilon \\ \Rightarrow & \frac{1}{n} \sum_{k=1}^{n} \left[M_k \left(S_k \left(\frac{|A_k(x)|}{\rho} \right) \right) \right]^{p_k} < \varepsilon. \end{aligned}$$

This implies that

$$\left\{ n \in N : \frac{1}{n} \sum_{k=1}^{n} \left[M_k \left(S_k \left(\frac{|A_k(x)|}{\rho} \right) \right) \right]^{p_k} \geq \varepsilon \right\} \subseteq A_\delta \in I.$$

This completes the proof.

(ii) Let $x = (x_k) \in W_0^I(\mathbf{M}, A, p) \cap W_0^I(\mathbf{S}, A, p)$. Then by the following inequality the result follows:

$$\begin{aligned} & \frac{1}{n} \sum_{k=1}^{n} \left[(M_k + S_k) \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} \\ \leq & D \frac{1}{n} \sum_{k=1}^{n} \left[M_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} + D \frac{1}{n} \sum_{k=1}^{n} \left[S_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k}. \end{aligned}$$

■

The proof of the following theorems can be established using standard techniques, so omitted.

Theorem 3.5. *Let* $0 < p_k \leq q_k$ *and* $\left(\frac{q_k}{p_k}\right)$ *is bounded, then*

$$W_0^I(\mathbf{M}, A, q) \subseteq W_0^I(\mathbf{M}, A, p).$$

Theorem 3.6. *For any two sequences* $p = (p_k)$ *and* $q = (q_k)$ *of positive real numbers, then the following holds:*

$$Z(\mathbf{M}, A, p) \cap Z(\mathbf{M}, A, q) \neq \phi, \textit{ for } Z = W^I, W_0^I, W_\infty^I \textit{ and } W_\infty^F.$$

Proposition 3.7. *The sequence spaces* $Z(\mathbf{M}, A, p)$ *are normal as well as monotone for the space* $Z = W_0^I$ *and* W_∞^I.

Proof. We give the prove of the proposition for $W_0^I(\mathbf{M}, A, p)$ only. Let $x = (x_k) \in W_0^I(\mathbf{M}, A, p)$ and $y = (y_k)$ be such that $|y_k| \leq |x_k|$ for all $k \in N$. Then for given $\varepsilon > 0$ we have

$$B = \left\{ n \in N : \frac{1}{n} \sum_{k=1}^{n} \left[M_k \left(\frac{|A_k(x)|}{\rho} \right) \right]^{p_k} \geq \varepsilon \right\} \in I.$$

Again the set

$$E = \left\{ n \in N : \frac{1}{n} \sum_{k=1}^{n} \left[M_k \left(\frac{|A_k(y)|}{\rho} \right) \right]^{p_k} \geq \varepsilon \right\} \subseteq B.$$

Hence $E \in I$ and so $y = (y_k) \in W_0^I(\mathbf{M}, A, p)$. Thus the space $W_0^I(\mathbf{M}, A, p)$ is normal. Also from the Lemma 1.5, it follows that $W_0^I(\mathbf{M}, A, p)$ is monotone. ∎

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