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On Characterization of Quasi-Complemented Almost Distributive Lattices

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Abstract. Characterization of quasi-complemented ADLs in terms of its α-ideals are studied. Some sufficient conditions of weakly disjunctive ADL to become quasi-complemented ADLs are derived.

Keywords: Almost Distributive Lattice; Prime ideal; Annihilator ideal; α-ideal; Weakly disjunctive ADL; Disjunctive ADL, quasi-complemented ADLs.

1. Introduction

The concept of an Almost Distributive Lattice (ADL) was introduced by U.M. Swamy and G.C. Rao in [16], as a common abstraction of existing lattice theoretic and ring theoretic generalizations of a Boolean algebra. They observered that the set of principal ideals of an ADL form a lattice under natural operations. Through this principal ideal lattice, several concepts from the lattice theory were extended to the class of Almost Distributive Lattices by several authors. Prominent among them are Stone ADLs [6], Alpha ideals in ADLs [13], Annulets in ADLs [14], Congruence Kernels in Pseudo-complimented Generalized Almost Distributive Lattices [11], Closure operators in ADLs [10], Heyting ADLs [4], L - ADLs [5], Normal ADLs [8], BL-ADLs [8], σ-Ideals of Almost Distributive Lattice [15], Pseudo-complemented Almost Distributive Lattices [17]. The concept of an α-ideal in ADLs and weakly disjunctive ADLs were introduced in [12] and a one-to-one correspondence between the set I_α(L) of all α-ideals of L and the set of all α-ideals of the lattice PI(L) of all principal ideals of L was established.

The concept of Quasi-complemented Almost Distributive Lattice was introduced [7] and some necessary and sufficient conditions for an Almost Distributive Lattice to become quasi-complemented Almost Distributive Lattice were proved. In this paper, we characterize the quasi-complemented ADL in terms of its α-ideals, annihilator ideals and dense elements. We derive some sufficient conditions for a weakly disjunctive ADL to become quasi-complemented ADLs.

2. Preliminaries

In this section, we give some important definitions and results taken from [3,7,12 ,16] for the sake of ready reference.

Definition 2.1. *An algebra $(L, \vee, \wedge, 0)$ of type $(2, 2, 0)$ is called an almost distributive lattice (ADL), if it satisfies the following axioms :*

(1) $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$,
(2) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$,
(3) $(a \vee b) \wedge b = b$,
(4) $(a \vee b) \wedge a = a$,
(5) $a \vee (a \wedge b) = a$,
(6) $0 \wedge a = 0$,

for all $a, b, c \in L$

In the following a partial order is defined on an ADL $(L, \vee, \wedge, 0)$.

Definition 2.2. *For any $a, b \in L$, we say that a is less than or equal to b and write $a \leq b$ if $a \wedge b = a$ or equivalently, $a \vee b = b$.*

It can be observed that an ADL L satisfies almost all the properties of a distributive lattice except possibly the right distributivity of $\vee$ over $\wedge$, commutativity of $\vee$, commutativity of $\wedge$. Any one of these properties make an ADL L, a distributive lattice.

Definition 2.3. *Let $(L, \vee, \wedge, 0)$ be an ADL. By an interval in L we mean the set $[a, b] := \{x \in L/a \leq x \leq b\}$, for some $a, b \in L$ with $a < b$. Every interval $[a, b]$ in an ADL is a bounded distributive lattice. An ADL $(L, \vee, \wedge, 0)$ is said to be relatively complemented if every interval [a,b], $a < b$ in L is a Boolean algebra.*

Theorem 2.4. *Let L be an ADL and $a, b, c \in L$. Then we have the following:*

(1) $a \vee b = a \Leftrightarrow a \wedge b = b$

(2) $a \vee b = b \Leftrightarrow a \wedge b = a$
(3) $a \wedge b = b \wedge a$, *whenever* $a \leq b$
(4) $\wedge$ *is associative in* L
(5) $a \wedge b \wedge c = b \wedge a \wedge c$
(6) $(a \vee b) \wedge c = (b \vee a) \wedge c$
(7) $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$
(8) $a \vee b = b \vee a$, *whenever* $a \wedge b = 0$
(9) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
(10) $a \wedge (a \vee b) = a, (a \wedge b) \vee b = b$ *and* $a \vee (b \wedge a) = a$
(11) $a \leq a \vee b$ *and* $a \wedge b \leq b$
(12) $a \wedge a = a$ *and* $a \vee a = a$
(13) $a \wedge 0 = 0$ *and* $0 \vee a = a \vee 0 = a$
(14) *If* $a \leq c$ *and* $b \leq c$, *then* $a \wedge b = a \wedge b$ *and* $a \vee b = a \vee b$
(15) $a \vee b = (a \vee b) \vee a$.

A non-empty subset I of an ADL L is called an ideal (filter) of L if $a \vee b \in I$ $(a \wedge b \in I)$ and $a \wedge x \in I$ $(x \vee a \in I)$, for any $a, b \in I$ and $x \in L$. If I is an ideal of L and $a, b \in L$, then $a \wedge b \in I \Leftrightarrow b \wedge a \in I$. The set I(L) of all ideals of L is a complete distributive lattice under set inclusion with the least element (0] and the greatest element L in which, for any $I, J, I \cap J$ is the infimum of I, J and the supremum is given by $I \vee J = \{i \vee j / i \in I, j \in J\}$. For any $a \in L, (a] = \{a \wedge x / x \in L\}$ is the principal ideal generated by a. Similarly, $[a) = \{x \vee a / x \in L\}$ is the principal filter generated by a. The set $PI(L)$ of all principal ideals of L is a sub lattice of $I(L)$. An ADL L with 0 is called a $*$-ADL if to each $x \in L$, there exists $y \in L$ such that $[x]^{**} = [y]^{*}$. An ADL L with 0 is a $*$-ADL if and only if to each $x \in L$, there exists $y \in L$ such that $x \wedge y = 0$ and $x \vee y \in D$. Every $*$-ADL possesses a dense element. A proper ideal P of L is said to be a prime ideal if for any $x, y \in L$, $x \wedge y \in P$ implies either $x \in P$ or $y \in P$. For any subset S of an ADL L, we define $S^{*} = \{x \in L / x \wedge a = 0, for all a \in S\}$. Then S^{*} is an ideal of L and is called the annihilator ideal of S. An ideal I of L is called an annihilator ideal if $I = I^{**}$ or equivalently, I $=S^{*}$, for some non-empty subset S of L. An ADL L with 0 is said to be disjunctive if $(a]^{*} = (b]^{*}$ implies $a = b$, for all $a, b \in L$. An element $a \in L$ is called dense if $(a]^{*}$ =(0]. The set of all dense elements in L is denoted by D and $DI(L)$ is the set of all dense ideals of L. The set D is a filter, whenever D is non-empty. An ideal I of an ADL L is called an α-ideal if $(x]^{**} \subseteq I$, for all $x \in I$. For any ideal I of L, the set $I^{e} = \{x \in L / [a]^{*} \subseteq [x]^{*}$, for some $a \in I\}$ is the smallest α-ideal containing I and hence I is an α-ideal if and only if $I = I^{e}$. The set $I_{\alpha}(L)$ of all α-ideals of L is a complete distributive lattice with respect to the operations $\wedge$ and $\tilde{\vee}$ given by $I \wedge J = I \cap J$ and $I \tilde{\vee} J = (I \vee J)^{e}$.

Lemma 2.5. *Let* L *be an ADL with 0 and* $a, b \in L$.*Then we have the following :*

(1) $(a] \vee (b] = (a \vee b] = (b \vee a]$

(2) $(a] \cap (b] = (a \wedge b] = (b \wedge a]$
(3) *For all* $x \in L, (x] = L$ *if and only if* x *is a maximal*
(4) *Every maximal element in* L *is dense element*
(5) *Every annihilator ideal in an* L *is an* α*-ideal*
(6) *For any two ideals* I, J *of* L*, then* $(I \vee J)^* = I^* \cap J^*$.

Theorem 2.6. *For any ideal* I *of an ADL* L*, the following are equivalent:*

(1) I *is an* α*-ideal*
(2) $I = \bigcup_{x \in I} [x]^{**}$
(3) *For any* $x, y \in L$, $[x]^* = [y]^*$ *and* $x \in I$ *implies* $y \in I$.

Theorem 2.7. *Let* I *be a proper ideal of an ADL* L *and* D*, a non-empty subset of* L *which is closed under* $\wedge$ *such that* $I \cap D = \emptyset$*. Then, there exists a prime ideal* P *of* L *such that* $I \subseteq P$ *and* $P \cap D = \emptyset$.

Theorem 2.8. *Let* I *be a proper* α*-ideal such that* $P \cap D = \emptyset$*. Then there exists a prime* α*-ideal* P *such that* $I \subseteq P$ *and* $P \cap D = \emptyset$.

Theorem 2.9. *Let* S *be any non-empty subset of* L *which is closed under* $\wedge$ *operation. Define,*$I = \{x \in L \ / \ x \wedge y = 0$*, for some* $y \in S\}$*. Then* I *is an* α*-ideal of* L.

Definition 2.10. *Let* L *be an ADL with 0. Then a proper ideal* I *of* L *is said to be semi-complemented if there exists an ideal* $J \neq (0]$ *such that* $I \cap J = (0]$.

Definition 2.11. *An ADL* L *with 0 is said to be weakly disjunctive ADL if* $(a]^* = (b]^*$ *implies* $(a] = (b]$*, for all* $a, b \in L$.

Theorem 2.12. [12] *Let* L *be an ADL with 0 in which* D *is non-empty. Then the following conditions are equivalent:*

(1) L *is a* $*$*-ADL and* $I^* \neq (0]$*, for any proper* $\alpha-$*ideal* I *of* L
(2) $I \cap D \neq \emptyset$*, for each* $I \in DI(L)$
(3) $I_\alpha(L)$ *is a semi complemented*
(4) $I_\alpha(L)$ *has unique dense element.*

Theorem 2.13. [12] *Let* L *be an ADL with 0. Then the following are equivalent:*

(1) L *is a weakly disjunctive ADL*
(2) *Every ideal of* L *is an* $\alpha-$*ideal*
(3) *Every prime ideal of* L *is an* $\alpha-$*ideal*

Definition 2.14. *An Almost Distributive Lattice L with 0 is called quasi-complemented ADL if for each $x \in L$,there is an element $y \in L$ such that $x \wedge y = 0$ and $x \vee y$ is a maximal. Here y is called a quasi-complement of x.*

Theorem 2.15. [7] *Let L be an ADL with a maximal element m. Then L is quasi-complemented if and only if every prime ideal in L is maximal ideal.*

Theorem 2.16. [7] *Let L be an ADL with 0 in which every prime ideal in L is an annihilator ideal. Then every prime ideal in L is maximal ideal.*

3. An α-ideal Characterization of Quasi-complemented Almost Distributive Lattices

In this section, we characterize the quasi-complemented ADL in terms of its α-ideals, annihilator ideals and dense elements. Also, we give some sufficient conditions for a weakly disjunctive ADL to become a quasi-complemented ADL. In the following, we prove certain properties of quasi-complemented ADLs.

Remark: Throughout the text, L denote an ADL with 0, unless otherwise mentioned.

Theorem 3.1. *Let L be quasi-complemented ADL. If a prime ideal P of L contains no dense element, then P is an $\alpha-$ideal.*

Proof. Let P be a prime ideal of L such that $P \cap D = \emptyset$. Let $a, b \in L$ such that $[a]^* = [b]^*$ and $a \in P$. Since L is quasi-complemented, there exists $x \in L$ such that $x \wedge a = 0$ and $x \vee a$ is a maximal. Thus $a \wedge x = 0$ and $a \vee x$ is a dense element in L. Since $P \cap D = \emptyset$, $a \vee x \notin P$ and hence $x \notin P$ because $a \in P$. Again, since $a \wedge x = 0$, $[a]^* = [b]^*$ and implies $b \wedge x = 0$. Therefore $b \in P$ since $x \notin P$. Hence by Theorem 2.6, P is an $\alpha-$ideal. ■

Corollary 3.2. *Let L be a quasi-complemented ADL. If no proper $\alpha-$ideal of L is a dense ideal, then every prime dense ideal of L contains a dense element.*

Proof. Let P be a prime dense ideal of L. Suppose $P \cap D = \emptyset$. Then, by Theorem 3.1, P is an $\alpha-$ideal. It follows that P is non-dense, which is a contradiction to our assumption. Hence $P \cap D \neq \emptyset$. Thus P contains dense element.

In the following, we give a necessary and sufficient condition for ADL to become quasi-complemented ADL. For this, first we need the following. ■

Lemma 3.3. *Let L be a quasi-complemented ADL in which no proper $\alpha-$ideal of L is a dense ideal. Then $I_\alpha(L)$ of all $\alpha-$ideal of L, is semi-complemented ADL.*

Proof. We have every quasi-complemented ADL is a $*$-ADL. By Theorem 2.12, we get $I_\alpha(L)$ is semi-complemented ADL. ■

Lemma 3.4. *If $I_\alpha(L)$ is semi complemented , then L is the only dense $\alpha-$ideal in L.*

Proof. Suppose $I_\alpha(L)$ is semi complemented. We know that L is a dense $\alpha-$ideal. Suppose there exists a proper $\alpha-$ideal I of L such that $I^* = (0]$. Since $I_\alpha(L)$ is semi complemented, there exists $J \in I_\alpha(L)$ and $J \neq (0]$ such that $I \cap J = (0]$. Then $J \subseteq I^*$ and hence $J = (0]$, which is a contradiction. Therefore, L is the only dense $\alpha-$ideal. ■

Now, we give the following result, which is consequence of Lemmas 3.3 and 3.4.

Corollary 3.5. *Let L be quasi-complemented ADL. Then L is the only dense $\alpha-$ideal.*

Now, we prove the following theorem

Theorem 3.6. *Let L be ADL with 0. then L is quasi-complemented if and only if every dense element is maximal and no proper α-ideal of L is dense.*

Proof. Suppose L is quasi-complemented ADL. Then, clearly every dense element is maximal. By Corollary 3.5, L is only the dense $\alpha-$ideal. Conversely suppose every dense element is a maximal and no proper α-ideal of L is dense. Let $x \in L$. Put I $= [x]^* \vee [x]^{**}$. Then we have I^e is an $\alpha-$ ideal in L. Further, $[x]^* \subseteq I \subseteq I^e$ and hence $[x]^* \subseteq I^e$. It follows that $[I^e]^* \subseteq [x]^{**}$ and $[x]^{**} \subseteq I \subseteq I^e$. Therefore $[I^e]^* \subseteq [x]^{***}$ and hence $[I^e]^* \subseteq [x]^* \cap [x]^{**} = (0]$. Hence I^e is a dense $\alpha-$ideal in L. Therefore, by hypothesis $I^e = L$. As $D \neq \emptyset$, choose $d \in D$ since $I^e = L$, $d \in I^e$ and hence there exists $t \in I$ such that $[t]^* \subseteq [d]^* = (0]$. Therefore $[t]^* = (0]$. As $t \in I = [x]^* \vee [x]^{**}$, we write $t = a \vee b$, where $a \in [x]^*$ and $b \in [x]^{**}$. Since $a \wedge b = 0$, $[a \vee b]^* = [t]^* = (0]$. Therefore $[a]^* \cap [b]^* = (0]$. It follows that $[a]^* \subseteq [b]^{**}$. As $b \in [x]^{**}$, we get $[b]^{**} \subseteq [x]^{**}$. Thus, we have $[a]^* \subseteq [x]^{**}$. On the other hand $a \in [x]^*$, implies $[x]^{**} \subseteq [a]^*$. Hence $[x]^{**} = [a]^*$. Thus for any $x \in L$, there exist $a \in L$ such that $[a]^* = [x]^{**}$. It follows that $x \wedge a = 0$ and $x \vee a$ is a dense element. Hence $x \wedge a = 0$ and $x \vee a$ is a maximal element. Thus L is quasi-complemented ADL. Now, we give a sufficient condition for an ADL to become a quasi-complemented ADL ■

Theorem 3.7. *Let L be an ADL. If every $\alpha-$ideal in L is a principal ideal and every dense element is maximal, then L is a quasi-complemented ADL.*

Proof. Let $x \in L$. Then $[x]^*$ being an annihilator ideal , is an $\alpha-$ideal. By hypothesis, $[x]^* = (a]$, for some $a \in L$. It follows that $[x]^{**} = [a]^*$. Since $x \in [x]^{**} = (a]^*$, $x \wedge a = 0$. Now, $[x \vee a]^* = [x]^* \cap [a]^* = [x]^* \cap [x]^{**} = (0]$. Therefore $x \wedge a = 0$ and $x \vee a$ is a dense. Thus $x \wedge a = 0$ and $x \vee a$ is a maximal. Hence L is quasi-complemented ADL. ■

Every annihilator ideal of L is an $\alpha-$ideal. But, an $\alpha-$ideal of L need not be an annihilator ideal. For example, a proper dense $\alpha-$ideal is not an annihilator ideal. Now, we give sufficient conditions for weakly disjunctive ADL to become a quasi-complemented ADL. For this, we need the following.

Theorem 3.8. *Let L be an ADL. If every dense ideal in L contains a dense element, then every $\alpha-$ideal in L is an annihilator ideal.*

Proof. Suppose every dense ideal in L contains a dense element. Let I be an $\alpha-$ideal. Since $[I \vee I^*]^* = I^* \cap I^{**} = (0]$, $I \vee I^*$ is a dense ideal. Therefore $I \vee I^*$ contains a dense element. Choose $t \in (I \vee I^*) \cap D$. Then $t = a \vee b$, for some $a \in I$, $b \in I^*$ and $[t]^* = (0]$. Now, $[a]^* \cap [b]^* = [a \vee b]^* = [t]^* = (0]$ and hence $[b]^* \subseteq [a]^{**}$. Now, let $x \in I^{**}$. Then $x \wedge b = 0$ since $b \in I^*$. Hence $x \in [b]^* \subseteq [a]^{**} \subseteq I$ since $a \in I$ and I is an $\alpha-$ideal. Thus $I^{**} \subseteq I$. Clearly $I \subseteq I^{**}$. Thus $I = I^{**}$. Therefore I is an annihilator ideal. ∎

In general, a prime ideal need not be an $\alpha-$ideal in an ADL. For, consider the following example

Example 3.9. Let A=$\{0, a\}$, $B = \{0, b_1, b_2\}$ be two discrete ADLs. Then ,$A \times B = \{(0,0), (0, b_1), (0, b_2), (a, 0), (a, b_1), (a, b_2)\}$ is an ADL under point wise operations of $\vee$ and $\wedge$. In this ADL, $S = \{(0,0), (0, b_1), (0, b_2), (a, b_1), (a, b_2)\}$ is a sub ADL and $I = \{(0,0), (0, b_1), (0, b_2)\}$ is a prime ideal but, not an $\alpha-$ideal since $[(0, b_1)]^{**} = S \nsubseteq I$. In the following theorem, we give a sufficient condition for a prime ideal in L to be an α-ideal.

Theorem 3.10. *Let L be an ADL. If a prime ideal P of L is a non-dense ideal, then P is an $\alpha-$ideal.*

Proof. Suppose P is a prime ideal of L such that $P^* \neq (0]$. Let $x \in P$. Since $P^* \neq (0]$, there exists $t \in P^*$ such that $t \neq 0$. Since $x \in P$ and $t \in P^*$, $t \wedge x = 0$ and hence $t \in [x]^*$. Now, let $s \in [x]^{**}$. Then $s \wedge t = 0$ and hence $s \wedge t \in P$. It follows that $s \in P$ or $t \in P$. If $t \in P$, then $t \in P \cap P^* = (0]$, which is a contradiction to our assumption. Therefore $s \in P$. Hence $[x]^{**} \subseteq P$. Thus P is an $\alpha-$ ideal. ∎

Theorem 3.11. *Let L be an ADL. If every proper $\alpha-$ideal of L is non dense, then any dense ideal of L contains a dense element.*

Proof. Let I be a dense ideal of L. Since $I \subseteq I^e$, $(I^e)^* \subseteq I^*$. Therefore I^{e^*} $= (0]$. Suppose $I^e \cap D = \emptyset$. Since I^e is an $\alpha-$ideal, by Theorem 2.11, there exists a prime $\alpha-$ideal P of L such that $I^e \subseteq P$ and $P \cap D = \emptyset$. Since $I^e \subseteq P$, therefore $P^* \subseteq I^{e^*}$. Hence $P^* = (0]$, which is a contradiction to our assumption. Therefore $I^e \cap D \neq (0]$. Hence I^e contains a dense element, say d. Then there exist $x \in I$ such that $[x]^* \subseteq [d]^*$. Hence $[x]^* = (0]$. Thus I contains a dense element. ■

Theorem 3.12. *Let L be weakly disjunctive ADL in which every dense ideal in L contains a dense element. Then L is quasi-complemented ADL.*

Proof. Suppose L is weakly disjunctive ADL with 0 in which every dense ideal in L contains dense element. Let P be a prime ideal of L such that P is not a maximal. Then there exists a maximal ideal Q in L such that $P \subset Q$. Therefore, there exists $x \in L$ such that $x \in Q$ and $x \notin P$. Since Q is a prime ideal, by Theorem 2.13, Q is an $\alpha-$ideal in L. Again, by Theorem 3.8, Q is an annihilator ideal. Therefore, $Q = Q^{**}$. Now, $x \in Q = Q^{**}$ and hence $x \wedge y = 0$, for all $y \in Q^*$. Hence $x \wedge y \in P$, for all $y \in Q^*$. So that $y \in P$, for all $y \in Q^*$. It folows that $Q^* \subseteq P$. But, we have $Q^* \subseteq P^*$, therefore $Q^* \subseteq P \cap P^* = (0]$. Hence $Q^* = (0]$. Therefore, $Q^{**} = (0]^* =$ L, which is a contradiction since Q is a maximal. Hence P is a maximal ideal. Therefore, by Theorem 2.15, L is quasi-complemented ADL ■

Finally, we give sufficient conditions for a weakly disjunctive ADL to become a quasi-complemented ADL.

Theorem 3.13. *Let L be weakly disjunctive ADL. Then the following statements are equivalent:*

(1) $I^* \neq (0]$*, for any proper* $\alpha-$*ideal* I *of* L

(2) $I \cap D \neq \emptyset$*, for any dense ideal* I *of* L

(3) *Every* $\alpha-$*ideal is an annihilator ideal*

(4) $I_\alpha(L)$ *is a semi complemented*

(5) $I_\alpha(L)$ *has unique dense element.*

Further any of the above conditions implies that L is quasi-complemented ADL

Proof. (1)⇒(2) follows by Theorem 3.11.

(2) ⇒ (3) follows by Theorem 3.8.

(3) ⇒ (4): Suppose every $\alpha-$ideal is an annihilator ideal. Let I be an inner element in $I_\alpha(L)$. Then I is an annihilator ideal. Therefore, $I = A^*$, for some non-empty subset A of L. Now, we have $I \cap I^* = (0]$. If $I^* = (0]$, then $A^{**} = (0]$. Hence $A^{***} = (0]^* =$ L. It follows that $A^* = L$. Therefore I = L, which is a contradiction. Hence $I^* \neq (0]$. Thus $I_\alpha(L)$ is semi-complemented.

(4) ⇒ (5) follows by Lemma 3.4.

(5) ⇒ (1): Suppose$I_\alpha(L)$ has unique dense element. Since L is dense $\alpha-$ideal, any proper $\alpha-$ideal is non dense. It follows that $I^* \neq (0]$, for any proper $\alpha-$ideal of L. Finally, by Theorem 3.12, any of the above conditions implies that L is quasi-complemented ADL. ■

Since every disjunctive ADL is weakly disjunctive ADL, we get the following

Corollary 3.14. *Let L be disjunctive ADL. Then the following statements are equivalent :*

(1) $I^* \neq (0]$, *for any proper* $\alpha-$*ideal* I *of* L

(2) $I \cap D \neq \emptyset$, *for any dense ideal* I *of* L

(3) *Every* $\alpha-$*ideal is an annihilator ideal*

(4) $I_\alpha(L)$ *is a semi complemented*

(5) $I_\alpha(L)$ *has unique dense element. Further any of the above conditions implies that L is quasi-complemented ADL.*

References

[1] G. Birkhoff, *Lattice Theory*, Amer. Math. Soc. Colloq. Publ. **XXV**, Providence, U.S.A., 1967.

[2] Y.S. Pawar, S.S. Khopade, α-ideals and annihilator ideals in 0-distributive lattices, *Acta Univ. Palacki. Olomuc., Fac.rer., Mathematica* **49** (1) (2010) 63–74.

[3] G.C. Rao, *Almost Distributive Lattice*, Ph.D. Thesis, Andhra University, Waltair, 1980.

[4] G.C. Rao, B. Assayee, Heyting almost distributive lattices, *Inter. J. Comput. Congition* **8** (3) (2010) 85–89.

[5] G.C. Rao, B. Assayee, L-Almost distributive lattices, *Asian-Eur J. Math.* **4** (1) (2011) 173–180.

[6] G.C. Rao, N.G. Rao, Stone almost distributive lattices, *Southeast Asian Bull. Math.* **27** (2003) 513–526

[7] G.C. Rao, N.G. Rao, A. Lakshmana, Quasi-complemented almost distributive lattice, *Southeast Asian Bull. Math.* **39** (3) (2015) 311–319.

[8] G.C. Rao, N.K. Kumar, BL-Almost distributive lattices, *Asian-Eur J. Math.* **5** (12) (2012) 1250022–1250029.

[9] G.C. Rao, N. Rafi, Normal almost distributive lattice, *Southeast Asian Bull. Math.* **32** (5) (2008) 831–841.

[10] G.C. Rao, N. Rafi, Closure operators in almost distributive lattices, *Inter. Math. Forum* **5** (19) (2010) 929–935.

[11] G.C. Rao, B. Ravi Kumar, A characterization of congruence kernels in psueudo-complmented generalized almost distributive lattice, *Inter. J. Algebra* **3** (16) (2009) 785–791.

[12] G.C. Rao, M. Sambasiva Rao, $\alpha-$ ideals and prime ideals in almost distributive lattice, *International Journal Algebra* **3** (5-7) (2009) 221–229.

[13] G.C. Rao, M. Sambasiva Rao, Alpha-ideals in almost distributive lattices, *International Journal of Contemp. Math. Science* **4** (9-12) (2009) 457–466.

[14] G.C. Rao, M. Sambasiva Rao, Annulets in almost distributive lattice, *European J. Pure and Applied Math.* 2 (1) (2009) 58–72.

[15] G.C. Rao, M. Sambasiva Rao, σ-Ideals of almost distributive lattices, *Asian-Eur J. Math.* **4** (4) (2012), 1250057 (10 pages).

[16] U.M. Swamy, G.C. Rao, Almost distributive lattice, *Aust. Math. Soc. Ser. A* **31** (1981) 77–91.