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A Note on Morita Equivalence Preserving Functor from the Category of Semigroups to the Category of Semirings

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Abstract. In this paper we have introduced a nice functorial connection between the category of semigroups and the category of semirings. Among other results we have proved that Morita equivalence is preserved by this functor.

Keywords: Power functor; Morita equivalence of semigroups (semirings); Morita context of semigroups (semirings).

1. Introduction and Preliminaries

In 1958 Morita [14] established the *Morita equivalence* theory for rings with identity which has been recognised as one of the most significant and fundamental tools in studying structure of rings. Later in 1972 both Knauer [13] and Banaschewski [1] independently transferred the ring theoretic approach to Monoids [12]. It is evident from various papers regarding *Morita equivalence* that this theory is an area of sustained research interest over decades. In 2011

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Katsov and Nam [11] generalized the *Morita theory* to semirings which was subsequently connected with *Morita context for semirings* by Sardar et.al. [16]. Sardar and Gupta also listed some properties that remain invariant under *Morita equivalence* of Semirings or Semigroups via Morita context [6, 7, 15, 5]. A recent work by Sardar, Gupta and Dey [3] described a nice interplay between various components of *Morita context* for semirings. In this paper we define a functor from the category of semigroups to the category of semirings and deduce that *Morita equivalence* is preserved by this functor.

Before presenting the main result in section 2, we recall some necessary preliminaries. Let $(S, +, \cdot)$ be a semiring and $a \in S$. Then a is called *k-regular* if there exist $x, y \in S$ such that $a + axa = aya$. If every element of the semiring S is k-regular then it is called *k-regular* [17]. If in the semiring $(S, +, \cdot)$, $(S, +)$ is a semilattice then the defining condition of k-regularity of an element $a \in S$ reduces to $a + axa = axa$ for some $x \in S$. If every element of a semiring S is *k-idempotent/ almost idempotent* i.e. $a + a^2 = a^2$ then it is called an *almost idempotent* semiring [21].

Suppose $(S, +, \cdot)$ is a semiring whose additive reduct is semilattice. Then

(i) S is called a *left k-semifield* if for all $a \in S$ and $b \in S'$, (if S contains additive identity 0 then $S' = S \setminus \{0\}$) there exists $u \in S$ such that $a + ub = ub$. Moreover if $a + bu = bu$ then it is called a *k-semifield* [18].

(ii) if every element $a \in S$ is *k-regular* satisfying the additional conditions $ax + ax^2a = ax^2a$ and $xa + ax^2a = ax^2a$ for some $x \in S$ then S is called a *completely k regular* semiring [18].

(iii) S is called a *left k-Clifford* semiring if S is a k-regular semiring and for all $a, b \in S$ there exists $x \in S$ such that $ab + xa = xa$. If in addition for each k-idempotent e and each $a \in S$ there is u in S such that $ae + eua = eua$ and $ea + aue = aue$ then S is called a *Clifford* semiring [19].

A nonempty set P is called a *right act over a monoid S*, written as P_S, if there is a mapping from: $P \times S \longrightarrow P$ defined by $(p, s) \longmapsto ps$ satisfying the properties

(i) $(ps_1)s_2 = p(s_1s_2)$ for all $p \in P, s_1, s_2 \in S$, and

(ii) $p1 = p$ for all $p \in P$.

Similarly we can define *left act over a monoid.* Also ${}_TP_S$ is called a *bi-act* if it is left T act and right S act with an additional property $(tp)s = t(ps)$ for all $t \in T, p \in P, s \in S$. Two monoids S and T are said to be *Morita equivalent* [12] if category of right (left) acts over S and category of right (left) acts over T are equivalent. A six tuple $< S, T, {}_TP_S, {}_SP'_T, \tau, \mu >$, where ${}_TP_S, {}_SP'_T$ are bi-acts over monoids S and T, $\mu :_T (P \otimes P')_T \longrightarrow T$ and $\tau :_S (P' \otimes P)_S \longrightarrow S$ are *bi-act morphisms* satisfying

(i) $\mu(q \otimes q'_1)q_1 = q\tau(q'_1 \otimes q_1)$, and

(ii) $q'\mu(q \otimes q'_1) = \tau(q' \otimes q)q'_1$, for all $q, q_1 \in P, q', q'_1 \in P'$.

is called a *Morita context for monoids.* S and T are said to be *Morita equivalent* if τ and μ are surjective. According to Golan [4], semiring is defined as

(i) $(R, +)$ is a commutative monoid.
(ii) $(R, \cdot)$ is a monoid.
(iii) Distributive property holds and
(iv) additive identity 0 is absorbing i.e. $r \cdot 0 = 0 \cdot r = 0$.

A semiring [9] is an algebra $(R, +, \cdot)$ where

(i) $(R, +)$ is a commutative semigroup
(ii) $(R, \cdot)$ is a semigroup
(iii) $\cdot$ distributes over $+$ from either sides.

Let $(Q, +)$ be additive commutative monoid. According to Katsov [11] R is a semiring with 0 and 1 with extra conditions

(i) $1_R p = p$.
(ii) $r 0_Q = 0_Q = 0_Q r$.

Then ${}_RQ$ is called a *left semimodule* over a semiring R if there is a mapping $R \times Q \longrightarrow Q$, $(r, q) \longmapsto rq$, satisfying

(i) $(rr_1)p = r(r_1 p)$
(ii) $(r + r_1)p = rp + r_1 p$
(iii) $r(p + p_1) = rp + rp_1$ for all $r,\ r_1 \in R,\ p, p_1 \in Q$.

In a similar manner we can define right R' semimodule $Q_{R'}$. Also ${}_RQ_{R'}$ is called a bi-semimodule if Q is a left R semimodule and a right R' semimodule with an additional condition

(iv) $(rp)r' = r(pr')$, for all $r \in R, p \in Q, r' \in R'$.

Two semirings R and R' are said to be Morita equivalent [11] if the category of right (left) R semimodules and the category of right (left) R' semimodules are equivalent. Let R and R' be two semirings, ${}_RQ_{R'}$ and ${}_{R'}Q'_R$ be two semimodules [11], $\theta :_{R'} (Q' \otimes Q)_{R'} \longrightarrow R'$ and $\phi :_R (Q \otimes Q')_R \longrightarrow R$ be two *bimodule morphisms* satisfying

(i) $\phi(q \otimes q_1')q_1 = q\theta(q_1' \otimes q_1)$ and
(ii) $q'\phi(q \otimes q_1') = \theta(q' \otimes q)q_1'$, for all $q, q_1 \in Q, q', q_1' \in Q'$.

Then the six tuple $< R, R', {}_RQ_{R'}, {}_{R'}Q'_R, \theta, \phi >$ is called a *Morita context for semirings* [16]. If in addition θ and ϕ are surjective then R and R' are said to be Morita equivalent semirings.

The class of semirings whose additive reduct is a semilattice has been a topic of sustained research interest which is evident from various publications such as [17, 18, 19, 20, 21, 8]. A rich source of example of such a class of semiring is the power semiring of a semigroup whose definition we recall below [20]. For a semigroup $(S, \cdot)$ if one defines on the power set $P(S)$ addition and multiplication as follows :

$$A + B := A \cup B \quad \text{and} \quad A \cdot B := \{a \cdot b : a \in A, b \in B\}$$

then $(P(S), +, \cdot)$ becomes a semiring whose additive reduct is a semilattice. This procedure of getting a semiring from a semigroup gives rise to correspondence, say, $P : Sgr \longrightarrow Sring$ where Sgr denotes the category of semigroups and $Sring$ denotes the category of semirings. In the next section P will be proved to be a functor (cf. Theorem 2.3(i)). The main purpose of this paper is, among other things, to establish the Morita equivalence preserving property of the functor P i.e., to prove that if S and T are two Morita equivalent monoids then $P(S)$ and $P(T)$ becomes Morita equivalent semirings (cf. Theorem 2.3(viii)).

2. Main Result

In this section we obtain our main result involving the function $P : Sgr \longrightarrow Sring$. At first we obtain two results to be used in the main result.

Proposition 2.1. *If ${}_TU_S$ is a bi-act then its image under $P : Sgr \longrightarrow Sring$ becomes a bi-semimodule viz., ${}_{P(S)}P(U)_{P(T)}$.*

Proof. Let ${}_TU_S$ is a bi-act. Then the power set $P(U)$, with $+$ defined as:

$A + B := A \cup B$ for all $A, B \in P(U)$, a commutative monoid. Further $P(T) \times P(U) \longrightarrow P(U)$, defined by $(T, Q) \longmapsto TQ := \{(t \cdot q) : t \in T, q \in Q\}$, is well defined as P is $T-S$ bi-act and makes $P(U)$ a left $P(T)$-semimodule because the following conditions can be easily verified by using the relevant definitions.

(i) $(T_1 + T_2)Q = T_1Q + T_2Q$,
(ii) $T_1(Q_1 + Q_2) = T_1Q_1 + T_1Q_2$,
(iii) $(T_1(T_2Q) = (T_1T_2)Q$,
(iv) $(\{1\}, Q) = Q$ (1 is the identity of T) for all $T_1, T_2, \in P(T)$ and Q_1, Q_2, $Q \in P(U)$.

In a similar fashion if we define $P(U) \times P(S) \longrightarrow P(U)$ by $(Q, S_1) \longmapsto QS_1 := \{qs_1 : q \in Q \subseteq U, s_1 \in S_1 \subseteq S\}$, then $P(U)$ becomes right $P(S)$ semimodule. In fact $P(U)$ becomes a $P(T) - P(S)$ bi-semimodule as for any $T_1 \in P(T)$, $Q \in P(U)$ and $S_1 \in P(S)$, $((T_1, Q), S_1) := \{((t_1q)s_1) : t_1 \in T_1, q \in Q, s_1 \in S_1\} = \{(t_1(qs_1)) : t_1 \in T_1, q \in Q, s_1 \in S_1\} = (T_1, (Q, S_1))$. [Second equality holds as P is a $T - S$ bi-act]. ∎

Proposition 2.2. *Suppose ${}_TU_S$ and ${}_SU'_T$ are two bi-acts over monoid such that $\tau :_S (U' \otimes_T U)_S \longrightarrow S$ is a surjective bi-act morphism. Then ${}_{P(T)}P(U)_{P(S)}$ and ${}_{P(S)}P(U')_{P(T)}$ are two bisemimodules where $\tilde{\theta} : (P(U') \otimes_{P(T)} P(U)) \longrightarrow P(S)$ is defined by $\tilde{\theta}(Q' \otimes Q) = \{\tau(q' \otimes q) : q' \in Q', q \in Q\}$ is a surjective bi-semimodule morphism.*

Proof. In view of Proposition 2.1, ${}_{P(T)}P(U)_{P(S)}$ and ${}_{P(S)}P(U')_{P(T)}$ are bisemimodules. Now let us define $\theta : (P(U') \times P(U)) \longrightarrow P(S)$ by $(Q', Q) \longmapsto Q'Q := \{\tau(q' \otimes_T q) : q' \in Q', q \in Q\}$.

Clearly θ is well defined. Now we prove that θ has the following properties:

(i) $\theta(Q'_1 + Q'_2, Q) = \theta(Q'_1, Q) + \theta(Q'_2, Q)$ for $Q'_1, Q'_2 \in P(U'), Q \in P(U)$.

(ii) $\theta(Q', Q_1 + Q_2) = \theta(Q', Q_1) + \theta(Q', Q_2)$ for $Q' \in P(U'), Q_1, Q_2 \in P(U)$.
(iii) $\theta(Q'T, Q) = \theta(Q', TQ)$ for $T \in P(T), Q \in P(U), Q' \in P(U')$.

In order to prove (1) we see that $\theta(Q'_1, Q) \subseteq \theta(Q'_1 + Q'_2, Q)$ as $Q'_1 + Q'_2 := Q'_1 \cup Q'_2$. Similarly $\theta(Q'_2, Q) \subseteq \theta(Q'_1 + Q'_2, Q)$, and so $\theta(Q'_1, Q) + \theta(Q'_2, Q) \subseteq \theta(Q'_1 + Q'_2, Q)$.

Conversely, let $x \in \theta(Q'_1 + Q'_2, Q)$. So $x = \tau(q' \otimes_T q)$ where $q' \in (Q'_1 \cup Q'_2)$ and $q \in Q$. Either $q' \in Q'_1$ and hence $x \in \theta(Q'_1, Q)$ or $q' \in Q'_2$ and $x \in \theta(Q'_2, Q)$. Thus $\theta(Q'_1 + Q'_2, Q) \subseteq \theta(Q'_1, Q) + \theta(Q'_2, Q)$. Hence $\theta(Q'_1 + Q'_2, Q) = \theta(Q'_1, Q) + \theta(Q'_2, Q)$.

Similarly we can prove (2). Now (3) follows as $x \in \theta(Q'T, Q) \iff x = \tau(q't \otimes_T q)$ for any $q \in Q, q' \in Q', t \in T$. $\iff x = \tau(q' \otimes_T tq) \iff x \in \theta(Q', TQ)$.

Hence θ is a $P(T)$ balanced map from $(P(U') \times P(U))$ to $P(S)$. Then there exists a unique morphism $\tilde{\theta} : (P(U') \otimes_{P(T)} P(U)) \longrightarrow P(S)$ such that $\tilde{\theta}(Q' \otimes Q) = \theta(Q', Q) = \{\tau(q' \otimes q) : q' \in Q', q \in Q\}$ where $P(U') \otimes P(U)$ is the tensor product of two bi-semimodules [11]. Since the tensor product $P(U') \otimes P(U)$ is actually the quotient of free monoid F over $P(U') \times P(U)$ by the congruence ρ, generated by

(i) $((Q'_1 + Q'_2, Q), ((Q'_1, Q) + (Q'_2, Q)))$ for $Q'_1, Q'_2 \in P(U'), Q \in P(U)$.
(ii) $((Q', Q_1 + Q_2), ((Q', Q_1) + (Q', Q_2)))$ for $Q' \in P(U'), Q_1, Q_2 \in P(U)$.
(iii) $((Q'T, Q), (Q', TQ))$ for $T \in P(T), Q \in P(U), Q' \in P(U')$.

i.e., $P(U') \otimes P(U) = F/\rho$, so $\tilde{\theta}$ satisfies following properties :

(i) $\tilde{\theta}((Q'_1 + Q'_2) \otimes Q) = \tilde{\theta}(Q'_1 \otimes Q) + \tilde{\theta}(Q'_2 \otimes Q)$.
(ii) $\tilde{\theta}(Q' \otimes (Q_1 + Q_2)) = \tilde{\theta}(Q' \otimes Q_1) + \tilde{\theta}(Q' \otimes Q_2)$.
(iii) $\tilde{\theta}(Q'T \otimes Q) = \tilde{\theta}(Q' \otimes TQ)$ Where $T \in P(T)$.

Making a note of the fact that τ is a bi-act morphism, from the definition of $\tilde{\theta}$ and properties shown above we can readily conclude that $\tilde{\theta}$ is a bi-semimodule morphism. In order to complete the proof we now prove the surjectivity of $\tilde{\theta}$:

Let $A \subseteq S$ and $a_i \in A$. As τ is surjective, there exist $q'_i \in Q'$ and $q_i \in Q$ such that $\tau(q'_i \otimes q_i) = a_i$. Now

$$\tilde{\theta}(\sum_i (\{q'_i\} \otimes \{q_i\}) = \sum_i \{\tau(q'_i \otimes q_i)\} = \sum_i \{a_i\} = \bigcup_i \{a_i\} = A.$$

Hence $\tilde{\theta}$ is surjective. ■

The following is our main result in which we show that $P : Sgr \longrightarrow Sring$ is a functor which among other things preserves Morita equivalence.

Theorem 2.3. *Let* **Sgr** *and* **Sring** *respectively denote the category of all semigroups and the category of all semirings. Let $P : Sgr \longrightarrow Sring$ be defined in such a way that for any $M \in ob(Sgr)$, for any $f \in Hom(M, N)$, $M, N \in ob(Sgr)$; $P(M)$ is the semiring with $+$ and $\cdot$ as*

$$A + B := A \cup B \quad \text{and} \quad A \cdot B := \{a \cdot b : a \in A, b \in B\}$$

and $Pf : P(M) \longrightarrow P(N)$ is the semiring morphism defined by $Pf(A) := \{f(a) : \ a \in A\}$. Then P

(i) *is a covariant functor which is faithful and injective on objects. For future reference this functor is called the power functor.*
(ii) *maps a regular semigroup to k-regular semiring,*
(iii) *maps a group to a k-semifield,*
(iv) *a completely regular semigroup to a completely k-regular semiring,*
(v) *maps a clifford semigroup to a k-clifford semiring,*
(vi) *maps a band to an almost idempotent semiring,*
(vii) *maps an ideal of any object S of* **Sgr** *to a k-ideal of $P(S)$ and*
(viii) *maps two Morita equivalent monoids to two Morita equivalent semirings with identity.*

Proof. (i) According to our discussion in introduction it is clear that $P(M)$ is an object in $Sring$ for any object $M \in Sgr$. Now suppose $M, N \in ob(Sgr)$ and $f \in Hom(M, N)$, the set of all semigroup morphisms from M to N. Then for any $A, B \in P(M)$, $Pf(A+B) := Pf(A \cup B) = \{f(x) : x \in A \cup B\} \subseteq Pf(A)$ or $Pf(B)$, whence $Pf(A+B) \subseteq Pf(A) + Pf(B)$. Also both $Pf(A)$ and $Pf(B)$ being subsets of $Pf(A+B)$, we have, $Pf(A) + Pf(B) \subseteq Pf(A+B)$. Thus $Pf(A+B) = Pf(A) + Pf(B)$.

Again we see that $Pf(A \cdot B) := \{f(x) : x \in A \cdot B\} = \{f(a \cdot b) : a \in A,\ b \in B\} = \{f(a) \cdot f(b) : a \in A;\ b \in B\} \subseteq Pf(A) \cdot Pf(B)$. Also $Pf(A) \cdot Pf(B) = \{f(a) : a \in A\} \cdot \{f(b) : b \in B\} = \{f(a) \cdot f(b) : a \in A, b \in B\} = \{f(a \cdot b) : a \in A, b \in B\} \subseteq Pf(A \cdot B)$. [As f is a semigroup morphism.] Hence $Pf(A \cdot B) = Pf(A) \cdot Pf(B)$. Thus $Pf \in Hom(P(M), P(N))$.

Let $f : A \longrightarrow A$ be an identity morphism in the category **Sgr**. Then $Pf : P(A) \longrightarrow P(A)$ is the identity morphism in the category **Sring**. Again let $S, T, K \in ob(Sgr)$ and $f \in Hom(T, K)$, $g \in Hom(S, T)$. Then for any $A \in P(S)$, $P(f \circ g)(A) = \{(f \circ g)(a) : a \in A\} = \{f(g(a)) : a \in A\} = (Pf \circ Pg)(A)$. This implies that $P(f \circ g) = Pf \circ Pg$. Thus $P : Sgr \longrightarrow Sring$ is a functor which is covariant. Let $f, g \in Hom(S, T)$ such that $Pf = Pg$. Then $Pf(\{a\}) = Pg(\{a\})$ for each $a \in S$, which implies that $\{f(a) : a \in S\} = \{g(a) : a \in S\}$, i.e. $f(a) = g(a)$ for all $a \in S$. Hence $f = g$. Consequently, the functor P is faithful. Finally let $S, T, \in ob(Sgr)$ such that $P(S) = P(T)$. Then $\{s\} \in P(T)$ for all $s \in S$, and $\{t\} \in P(S)$ for all $t \in T$. Thus $s \in T$ for each s and $t \in S$ for each t. Hence $S = T$. Hence the functor P is injective on objects.

(ii) This is a part of Theorem 3.1 of [20].

(iii) This is a part of Proposition 2.2.3 of [2].

(iv) This is a part of Proposition 3.1.3 of [2].

(v) This is a part of Proposition 4.1.7 of [2].

(vi) This is a part of Proposition 3.1 of [21].

(vii) This is a part of Proposition 1.2.4 of [2].

(viii) Since S and T are Morita equivalent monoids. Then there is a Morita context $< S, T, {}_T P_S, {}_S P'_T, \tau, \mu >$ of monoids [16], where $\tau :_S (P' \otimes_T P)_S \longrightarrow$ S and $\mu :_T (P \otimes_S P')_T \longrightarrow T$ are surjective bi-act morphisms. For convenience let us write $\tau(q' \otimes q) = (q', q)$ and $\mu(q \otimes q') = [q, q']$. In view of Proposition 2.1,

${}_{P(T)}P(U)_{P(S)}$ and ${}_{P(S)}P(U')_{P(T)}$ are bisemimodules and in view of Prop. 2.2, $\tilde{\theta}$: $P(U')\otimes_{P(T)}P(U) \longrightarrow P(S)$ and $\tilde{\phi}$: $P(U)\otimes P(U') \longrightarrow P(T)$ denoted respectively by $\tilde{\theta}(Q'_1, Q) = (Q'_1, Q)$ and $\tilde{\phi}(Q, Q'_1) = [Q, Q'_1]$ are surjective bisemimodule morphisms satisfying the properties:

(a) $[Q, Q'_1]Q_1 = Q(Q'_1, Q_1)$.
(b) $Q'[Q, Q'_1] = (Q', Q)Q'_1$.

where $Q', Q'_1 \in P(U')$, and $Q, Q_1 \in P(U)$. These properties follow from the properties:

(a) $[q, q'_1]q_1 = q(q'_1, q_1)$.
(b) $q'[q, q'_1] = (q', q)q'_1$.

obtained previously. So we obtain the Morita context $< P(S), P(T), {}_{P(T)}P(U)_{P(S)}, {}_{P(S)}P(U')_{P(T)}, \tilde{\theta}, \tilde{\phi} >$ where $\tilde{\theta}$ and $\tilde{\phi}$ are surjective. Consequently, $P(S)$ and $P(T)$ are Morita equivalent semirings with identity (cf. Theorem 4.8 of [16]). This completes the proof. ■

The functor is neither full (and hence not an embedding) nor surjective on objects which is evident from the following examples.

Example 2.4. Let S and T be two semigroups. Let $\Psi : P(S) \longrightarrow P(T)$ be defined by $\Psi(A) = \emptyset$ for all $A \in P(S)$. Then Ψ is a semiring morphism but not of the form Pf. Hence the functor P is not full.

Example 2.5. Let $U = \{0, 1, 2\}$. Let us define $u+v := min\{u, v\}$ and $u \cdot v := u$, for all $u, v \in U$. Then U is a semiring whose additive reduct is a semilattice but not power of a semigroup as its number of elements $3 \neq 2^n$, for any positive integer n. Thus the functor is not surjective on objects.

3. Concluding Remark

Though the converse of each of (ii),(iii),(iv),(v),(vi),(vii) of Theorem 2.3 is true (cf. Respective results cited from [20], [21] or [2]), but we have not been able to prove or disprove the converse of (viii) of Theorem 2.3, which is "Suppose S and T are two monoids such that the power semirings $P(S)$ and $P(T)$ are Morita equivalent. Then S and T are Morita equivalent."

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