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On Languages Defined by Generalized Principal Right Congruences

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Dedicated to the memory of Professor Yuqi Guo (1940–2019)

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Abstract. The study of languages defined by generalized principal right congruences on a free monoid generated by a finite alphabet was initiated by Prodinger in 1980. In this paper, we introduce some kinds of semifilters related to suffix-closed languages and investigate languages defined by generalized principal right congruences associated with these semifilters. As an application, a characterization of regular languages is reobtained.

Keywords: Semifilters; Suffix-closed languages; Regular languages.

1. Introduction

It is well known that the class of *regular languages* forms an important class of languages in theoretical computer science. A language over a finite alphabet is *regular* if it can be accepted by a *finite states automaton*. Furthermore, regular languages also have a lot of remarkable algebraic characterizations. In particular, regular languages can be characterized by using the finiteness of the indexes of *principal right congruences* determined by themselves. In view of this point, we now investigate and generalize regular languages by applying generalizing principal right congruences.

This idea was realized first by Prodinger in [4], he explored a generalization model of principal right congruences by using so-called *left divisible semifilters*. By applying generalized principal right congruences determined by left divisible semifilters, Prodinger generalized regular languages over a finite alphabet.

In [4], Prodinger pointed out that the *cofinite semifilter* seemed to be the most interesting semifilter and proposed a problem related to this semifilter. In 1983, Guo-Wang-Li [1] solved this problem. In fact, they showed that regular languages over a finite alphabet can be characterized by using the finiteness of the index of a right congruence determined by the cofinite semifilter.

In this paper, we introduce some kinds of new semifilters related to suffix-closed languages and investigate languages defined by generalized principal right congruences associated with these semifilters. As an application, a characterization of regular languages is reobtained.

2. Preliminaries

Throughout this paper, X is a finite nonempty set that is called a *finite alphabet* in which any element is called *a letter* over X, and X^* always denotes the free monoid generated by X. Moreover, $w \in X^*$ and $L \subseteq X^*$ are called *a word* and *a language* over X, respectively. In particular, the identity of X^* is called the *empty word* over X and denoted by 1. The length of $w \in X^*$ is the number of letters appearing in w and is denoted by $|w|$.

Let S be a monoid, $L, K \subseteq S$ and $z \in S$. Then, we denote it by

$$z^{-1}L = \{w \in S \mid zw \in L\},\ Lz^{-1} = \{w \in S \mid wz \in L\}.$$

Moreover, we use $\overline{L}$ to denote the complement of L in S. We also denote the set of finite subsets of S by $\mathcal{F}(S)$, and the complement of the symmetric difference of L and K by $L \circ K$, respectively. Formally, we have

$$\mathcal{F}(S) = \{F \subseteq S \mid F \text{ is finite}\}, \quad L \circ K = (L \cap K) \cup \overline{L \cup K}.$$

Now, we recall some concepts dealing with *semifilters* on a monoid, which play central role in our discussions. Let S be a monoid, 2^S the power set of S and $\mathscr{L} \subseteq 2^S$. Then $\mathscr{L}$ is called a *semifilter* on S if the following hold:

(1) $S \in \mathscr{L}$,

(2) $(\forall A, B \in \mathscr{L}) \quad A \circ B \in \mathscr{L}$.

A semifilter $\mathscr{L}$ on S is called *left divisible* if

$$(\forall A \in \mathscr{L})(\forall z \in S) \quad z^{-1}A \in \mathscr{L}.$$

The followings are examples of semifilters on a monoid S.

(1) $\{S\}$.

(2) $\mathscr{C}(S) = \{L \subseteq S \mid \overline{L} \text{ is finite }\}$, $\mathscr{C}(S)$ is called the *cofinite semifilter* on S.

(3) $\mathscr{L}_M(S) = \{L \subseteq S \mid \overline{L} \subseteq M\}$ for a given $M \subseteq S$.
(4) $\mathscr{L}_{\mathcal{F}(S),M}(S) = \{L \subseteq S \mid (\exists F \in \mathcal{F}(S))\ \overline{L} \subseteq FM\}$ for a given $M \subseteq S$.

It is easy to see that $\{X^*\}$ and $\mathscr{C}(X^*)$ are left divisible semifilters on X^*. Moreover, it is clear to see that $\mathscr{L}_M(S) \subseteq \mathscr{L}_{\mathcal{F}(S),M}(S)$ for any $M \subseteq S$.

Now, we recall Prodinger's generalization model of principal right congruences by using left divisible semifilters on X^*. Recall that an equivalence ρ on X^* is called a *right congruence* if $x \,\rho\, y$ implies that $xz \,\rho\, yz$ for all $x, y, z \in X^*$. The index of an equivalence ρ is the number of ρ-classes of X^*. For convenience, we denote the class of regular languages over X by $\mathscr{R}(X)$ or $\mathscr{R}$ if no confusion arises. For any $L \subseteq X^*$, the relation $P_L^{(r)}$ is defined as follows:

$$x \, P_L^{(r)} \, y \text{ if and only if } P_L^{(r)}(x, y) = X^*,$$

where

$$P_L^{(r)}(x, y) = \{v \in X^* \mid xv \in L \text{ if and only if } yv \in L\}.$$

It can be proved that $P_L^{(r)}$ is a right congruence on X^*, which is called the *principal right congruence* on X^* determined by L. The following characterizations of regular languages are well-known.

Theorem 2.1. [3, 5] *Let $L \subseteq X^*$. Then L is regular if and only if the index of $P_L^{(r)}$ is finite.*

Let $\mathscr{L}$ be a left divisible semifilter on X^* and $L \subseteq X^*$. Define

$$x \, P_{\mathscr{L},L}^{(r)} \, y \text{ if and only if } P_L^{(r)}(x, y) \in \mathscr{L}.$$

Then from [4], $P_{\mathscr{L},L}^{(r)}$ is a right congruence on X^*. Furthermore,

$$P_L^{(r)} = P_{\{X^*\},L}^{(r)} \subseteq P_{\mathscr{L},L}^{(r)}$$

for any $L \subseteq X^*$. Thus, the principal right congruences determined by languages are generalized by using the left divisible semifilters on X^*. Furthermore, if we denote

$$\mathscr{R}_{\mathscr{L}}^{(r)} = \{L \subseteq X^* \mid \text{the index of } P_{\mathscr{L},L}^{(r)} \text{ is finite}\},$$

then by Theorem 2.1, $\mathscr{R} \subseteq \mathscr{R}_{\mathscr{L}}^{(r)}$. Thus, regular languages are generalized by this method.

In [4], Prodinger proposed the following question: Does $\mathscr{R} = \mathscr{R}_{\mathscr{C}(X^*)}$? Guo-Wang-Li [1] gave a positive answer for this question. In this paper, we mainly concentrate on the semifilters $\mathscr{L}_M(X^*)$ and $\mathscr{L}_{\mathcal{F}(X^*),M}(X^*)$, where $M \subseteq X^*$. The following result gives some elementary properties of $\mathscr{L}_{\mathcal{F}(S),M}(S)$ and $\mathscr{L}_M(S)$ for a 1-free monoid S and a given $M \subseteq S$. A monoid S with identity 1 is 1-*free* if $xy = 1$ implies that $x = y = 1$ for all $x, y \in S$. Obviously, X^* is 1-free.

Proposition 2.2. *Let S be a 1-free monoid and $M \subseteq S$. Then, the following statements hold:*

(1) $\mathscr{L}_{\mathcal{F}(S),M}(S) = 2^S$ *if and only if* $M \in \mathscr{L}_{\mathcal{F}(S),M}(S)$;
(2) $\mathscr{L}_M(S) = 2^S$ *if and only if* $M = S$;
(3) $\mathscr{L}_{\mathcal{F}(S),M}(S) = \{S\}$ (*resp.* $\mathscr{L}_M(S) = \{S\}$) *if and only if* $M = \emptyset$;
(4) $\mathscr{C}(S) \subseteq \mathscr{L}_{\mathcal{F}(S),M}(S)$ *if and only if* $1 \in M$;
(5) $\mathscr{L}_{\mathcal{F}(S),M}(S) \subseteq \mathscr{C}(S)$ *if and only if* $M \in \mathcal{F}(S)$.

Proof. (1) The necessity part is trivial. Now, let $L \in 2^S$ and $\overline{M} \subseteq FM$ for some $F \in \mathcal{F}(S)$. Take $F \cup \{1\} = F' \in \mathcal{F}(S)$. Then, $\overline{L} \subseteq S = F'M$, which implies that $L \in \mathscr{L}_{\mathcal{F}(S),M}(S)$.

(2) If $M = S$, then $\mathscr{L}_M(S) = \{L \subseteq S \mid \overline{L} \subseteq S\} = 2^S$. Conversely, if $M \neq S$, then $M \notin \mathscr{L}_M(S)$, since $\overline{M}$ is not a subset of M.

(3) Clear.

(4) Let $\mathscr{C}(S) \subseteq \mathscr{L}_{\mathcal{F}(S),M}(S)$. Then, $S \setminus \{1\} \in \mathscr{L}_{\mathcal{F}(S),M}(S)$ and so $\overline{S \setminus \{1\}} \subseteq FM$ for some $F \in \mathcal{F}(S)$. This yields that $1 \in FM$ and $1 \in M$, since S is 1-free. Conversely, if $1 \in M$ and $L \in \mathscr{C}(S)$, then $\overline{L}$ is finite and $\overline{L} \subseteq \overline{L}M$. This shows that $L \in \mathscr{L}_{\mathcal{F}(S),M}(S)$.

(5) The necessity follows from the fact $\overline{M} \in \mathscr{L}_{\mathcal{F}(S),M}(S)$. The sufficiency is clear. ■

3. Some Semifilters Related to Suffix-Closed Languages

In this section, we consider some properties of several kinds of semifilters related to suffix-closed languages over X. For the sake of convenience, for $M \subseteq X^*$, in the sequel, we shall use $\mathscr{C}$, $\mathscr{L}_M$ and $\mathscr{L}_{\mathcal{F},M}$ to denote $\mathscr{C}(X^*)$, $\mathscr{L}_M(X^*)$ and $\mathscr{L}_{\mathcal{F}(X^*),M}(X^*)$, respectively. Recall that a language L over X is called *prefix-closed* (resp. *suffix-closed*) if $Lz^{-1} \subseteq L$ (resp. $z^{-1}L \subseteq L$) for any $z \in X^*$. We denote the set of prefix-closed languages and suffix-closed languages over X by $\mathbb{P}(X^*)$ and $\mathbb{S}(X^*)$, respectively. In general, $\mathscr{L}_M$ and $\mathscr{L}_{\mathcal{F},M}$ may be not left divisible. However, we have the following result.

Proposition 3.1. *Let* $M \subseteq X^*$. *Then* $\mathscr{L}_M$ *is left divisible if and only if* $M \in \mathbb{S}(X^*)$.

Proof. Let $\mathscr{L}_M$ be left divisible and $u \in X^*$. Then, $\overline{M} \in \mathscr{L}_M$ and so $u^{-1}\overline{M} \in \mathscr{L}_M$. This implies that $u^{-1}M = \overline{u^{-1}\overline{M}} \subseteq M$. Thus, $M \in \mathbb{S}(X^*)$. Conversely, let $A \in \mathscr{L}_M$, $M \in \mathbb{S}(X^*)$ and $u \in X^*$. Then, $\overline{A} \subseteq M$ and $u^{-1}M \subseteq M$. This yields that $\overline{u^{-1}A} = u^{-1}\overline{A} \subseteq u^{-1}M \subseteq M$, that is, $u^{-1}A \in \mathscr{L}_M$. This shows that $\mathscr{L}_M$ is left divisible. ■

Proposition 3.2. *Let* $\emptyset \neq M \subseteq X^*$. *If* $\mathscr{L}_{\mathcal{F},M}$ *is left divisible, then* $1 \in M$.

Proof. Let $\mathscr{L}_{\mathcal{F},M}$ be left divisible and $z \in M$. Then $\overline{M} \in \mathscr{L}_{\mathcal{F},M}$ and $z^{-1}\overline{M} \in \mathscr{L}_{\mathcal{F},M}$. This implies that $1 \in z^{-1}M = \overline{z^{-1}\overline{M}} \subseteq F'M$ for some $F' \in \mathcal{F}(X^*)$. Therefore, $1 \in M$. ■

Remark 3.3. The converse of Proposition 3.2 is not true. For example, let $a \in X$ and $M = \{a^{\frac{n(n+1)}{2}+1} \mid n \geq 1\} \cup \{1\}$. Then $\mathscr{L}_{\mathcal{F},M}$ is not left divisible. In fact, in this case, $\overline{M} \in \mathscr{L}_{\mathcal{F},M}$. If $\mathscr{L}_{\mathcal{F},M}$ is left divisible, then $a^{-1}\overline{M} \in \mathscr{L}_{\mathcal{F},M}$. Hence, there exists $F \in \mathcal{F}(X^*)$ such that $a^{-1}M = \overline{a^{-1}\overline{M}} \subseteq FM$. Denote $T = max\{|f| \mid f \in F\}$. Take $n \geq T+2$. Then, $a^{\frac{n(n+1)}{2}} \in a^{-1}M \subseteq FM$. This implies that $a^{\frac{n(n+1)}{2}} = fm$ for some $f \in F$ and $m = a^{\frac{s(s+1)}{2}+1} \in M$ such that $s < n$. But this yields that

$$|f| = \frac{n(n+1)}{2} - |m| \geq \frac{n(n+1)}{2} - \frac{n(n-1)}{2} - 1 = n - 1 \geq T + 1.$$

A contradiction.

Proposition 3.4. *If $M \in \mathbb{S}(X^*)$, then $\mathscr{L}_{\mathcal{F},M}$ is left divisible.*

Proof. Let $L \in \mathscr{L}_{\mathcal{F},M}$ and $z \in X^*$. Then, there exists $F \in \mathcal{F}(X^*)$ such that $\overline{L} \subseteq FM$. We assert that $\overline{z^{-1}L} \subseteq F'M$, where $z^{-1}F \cup \{1\} = F' \in \mathcal{F}(X^*)$. In fact, if $w \notin z^{-1}L$, then $zw \in \overline{L}$. This implies that $zw \in FM$. Let $zw = fm$ for some $f \in F$ and $m \in M$. If w is a suffix of m, then by hypothesis, $w \in M \subseteq F'M$. Otherwise, there exists $f' \in X^*$ such that $w = f'm$ and $zf' = f \in F$, this implies that $w \in (z^{-1}F)M \subseteq F'M$. ∎

Remark 3.5. On the other hand, if $M \subseteq X^*$ and $\mathscr{L}_{\mathcal{F},M}$ is left divisible, we claim that M may be not in $\mathbb{S}(X^*)$. In fact, by (4) and (5) of Proposition 2.2, $1 \in M$ and $M \in \mathcal{F}(X^*)$ if and only if $\mathscr{L}_{\mathcal{F},M} = \mathscr{C}$. In this case, $\mathscr{L}_{\mathcal{F},M}$ is left divisible but M may not be in $\mathbb{S}(X^*)$.

Proposition 3.6. *Let $M \in \mathbb{S}(X^*)$. Then $\mathscr{L}_{\mathcal{F},M} = 2^{X^*}$ if and only if $M = X^*$.*

Proof. The sufficiency follows from (1) of Proposition 2.2. Now, let $M \neq X^*$ and $w \in \overline{M}$. Since $M \in \mathbb{S}(X^*)$, we have $X^*w \subseteq \overline{M}$. On the other hand, by hypothesis, $\overline{M} \subseteq FM$ for some $F \in \mathcal{F}(X^*)$. This shows that $X^*w \subseteq FM$. Let $T = max\{|f| \mid f \in F\}$ and $u \in X^*$ such that $|u| > T+1$. Then, $uw = fm$ for some $f \in F$ and $m \in M$. Observe that $|w| > |m|$ (otherwise, $m \in X^*w \subseteq \overline{M}$), it follows that $f = uf' \in F$ for some $f' \in X^*$. A contradiction. ∎

4. $\mathscr{R}_{\mathscr{C}}^{(r)}$, $\mathscr{R}_{\mathscr{L}_M}^{(r)}$ and $\mathscr{R}_{\mathscr{L}_{\mathcal{F},M}}^{(r)}$

In this section, we shall explore the relationship among $\mathscr{R}, \mathscr{R}_{\mathscr{C}}^{(r)}, \mathscr{R}_{\mathscr{L}_M}^{(r)}$ and $\mathscr{R}_{\mathscr{L}_{\mathcal{F},M}}^{(r)}$, where $M \in \mathbb{S}(X^*)$. From the previous sections, we know that $\mathscr{C}$, $\mathscr{L}_M$ and $\mathscr{L}_{\mathcal{F},M}$ are all left divisible semifilters on X^*. For our purpose, we need the following two lemmas which can be found in Guo-Wang-Shum [2].

Lemma 4.1. *Let $L \in \mathbb{P}(X^*)$ and L be infinite. Then there exists*

$$C = \{1, a_1, a_1a_2, \cdots, a_1a_2\cdots a_n, \cdots\} \subseteq L$$

such that C is infinite, where $a_i \in X$.

To give the next lemma, we need the following *alphabetic order* "$\leq$" on X^*: For two words u and v with different lengths, $u < v$ if $|u| < |v|$. For the two words with the same length, the order is the lexicographic order.

Lemma 4.2. *Let ρ be a right congruence on X^* and $\{L_i \mid i \in I\}$ be the set of all ρ-classes. Then,*

$$S = \{s_i \mid s_i \text{ is the least element in } L_i \text{ with respect to the order } \leq,\ i \in I\}$$

is a prefix-closed language. In this case, S is called the least cross-section of ρ.

Proposition 4.3. *Let $M \in \mathbb{S}(X^*)$. Then $\mathscr{R}^{(r)}_{\mathscr{L}_{\mathcal{F},M}} = \mathscr{R}^{(r)}_{\mathscr{L}_M}$.*

Proof. Observe that $\mathscr{L}_M \subseteq \mathscr{L}_{\mathcal{F},M}$, it follows that $\mathscr{R}^{(r)}_{\mathscr{L}_M} \subseteq \mathscr{R}^{(r)}_{\mathscr{L}_{\mathcal{F},M}}$. Conversely, if $L \in \mathscr{R}^{(r)}_{\mathscr{L}_{\mathcal{F},M}} \cap \overline{\mathscr{R}^{(r)}_{\mathscr{L}_M}}$, then the least cross-section (see Lemma 4.1) S of $P^{(r)}_{\mathscr{L}_M,L}$ is infinite. By Lemma 4.2, there exists

$$C = \{1, a_1, a_1a_2, \cdots, a_1a_2\cdots a_n, \cdots\} \subseteq S$$

such that C is infinite, where $a_i \in X$. Since $L \in \mathscr{R}^{(r)}_{\mathscr{L}_{\mathcal{F},M}}$, there exist two distinct elements $x, y \in C$ such that $x\, P^{(r)}_{\mathscr{L}_{\mathcal{F},M},L}\, y$. Therefore, $\overline{P^{(r)}_L(x,y)} \subseteq FM$ for some $F \in \mathcal{F}(X^*)$. Denote $T = max\{|f| \mid f \in F\}$ and take $u \in X^*$ satisfying $|u| > T$. We assert that $\overline{P^{(r)}_L(xu,yu)} \subseteq M$. In fact, if $v \in \overline{P^{(r)}_L(xu,yu)}$, then $uv \in \overline{P^{(r)}_L(x,y)} \subseteq FM$. This yields that $uv = fm$ for some $f \in F$ and $m \in M$. Since $|u| > |f|$, v is a suffix of m. Note that $M \in \mathbb{S}(X^*)$, $v \in M$. Hence, $xu\, P^{(r)}_{\mathscr{L}_M,L}\, yu$.

Without loss of generality, we let $x < y$, $y = a_1a_2\cdots a_t$ and $u = a_{t+1}\cdots a_{t+T+1}$. Then, by the above discussions, $xu\, P^{(r)}_{\mathscr{L}_M,L}\, yu$ and $yu \in C \subseteq S$. In view of the definition of S, we have $xu \geq yu$. This implies that $y \leq x$. A contradiction. ∎

Remark 4.4. In general, $P^{(r)}_{\mathscr{L}_{\mathcal{F},M},L} \neq P^{(r)}_{\mathscr{L}_M,L}$. For example, let $X = \{a\}$ and $M = \{1, a\}$. Then,

$$\mathscr{L}_M = \{K \subseteq X^* \mid \overline{M} \subseteq K\} = \{a^2a^*, a^*, a^+, a^*\backslash\{a\}\}.$$

By Proposition 2.2, $\mathscr{L}_{\mathcal{F},M} = \mathscr{C}$. Let $L = \{a^2, a^3\}$ and $F = \{1, a, a^2, a^3\} \in \mathcal{F}(X^*)$. Obviously, $P^{(r)}_L(1, a^2) = \overline{F} \notin \mathscr{L}_M$ and $P^{(r)}_L(1, a^2) \in \mathscr{L}_{\mathcal{F},M}$.

As an application of Proposition 4.3, we can reobtain the main result of [1] which gives a positive answer of the question proposed in [4].

Corollary 4.5. $\mathscr{R}^{(r)}_{\mathscr{C}} = \mathscr{R}$.

Proof. Let $M = \{1\}$. Then we have $\mathscr{L}_M = \{X^+, X^*\}$ and $\mathscr{L}_{\mathcal{F},M} = \mathscr{C}$. Observe that $P^{(r)}_L = P^{(r)}_{\mathscr{L}_M,L} \cap \sim^\circ_L$ for each $L \subseteq X^*$, where

$$\sim^\circ_L = \{(x, y) \in X^* \times X^* \mid x \in L \text{ if and only if } y \in L\}.$$

By Proposition 4.3, $\mathscr{R}^{(r)}_{\mathscr{C}} = \mathscr{R}^{(r)}_{\mathscr{L}_M} = \mathscr{R}^{(r)}_{\{X^*\}} = \mathscr{R}$. ∎

Finally, we consider the relationship between $\mathscr{R}$ and $\mathscr{R}^{(r)}_{\mathscr{L}_M}$. To this aim, the following result is needed.

Lemma 4.6. [6, Lemma 3.15] *Let $L \subseteq X^*$ and L be infinite. Then, there exists $L' \subseteq L$ such that $L' \notin \mathscr{R}$.*

Proposition 4.7. *Let $M \in \mathbb{S}(X^*)$. Then $\mathscr{R}^{(r)}_{\mathscr{L}_M} = \mathscr{R}$ if and only if $M \in \mathcal{F}(X^*)$.*

Proof. If $M \notin \mathcal{F}(X^*)$, then by Lemma 4.6, there exists $L \subseteq M$ such that $L \notin \mathscr{R}$. Since $\overline{M} \subseteq P^{(r)}_L(x, y)$ for every $x, y \in X^*$, we have $P^{(r)}_L(x, y) \in \mathscr{L}_M$ for every $x, y \in X^*$. This implies $L \in \mathscr{R}^{(r)}_{\mathscr{L}_M}$. A contradiction. Conversely, let $M \in \mathcal{F}(X^*)$. Observe that $M \in \mathbb{S}(X^*)$, $1 \in M$. By Proposition 2.2, $\mathscr{L}_{\mathcal{F},M} = \mathscr{C}$. By Proposition 4.3 and Corollary 4.5, we have $\mathscr{R}^{(r)}_{\mathscr{L}_M} = \mathscr{R}^{(r)}_{\mathscr{L}_{\mathcal{F},M}} = \mathscr{R}$. ∎

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