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The K.Banhatti Indices of Certain Graphs

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Abstract. Topological indices are numbers related to molecular graphs that catch symmetry of molecular structures and give it a scientific dialect to foresee properties, such as boiling points, viscosity, the radius of gyrations and so on. In this paper, we aim to compute the general formula for first and second K.Banhatti indices, first and second k hyper-Banhatti indices for graph gluing of web graph and also subdivision and semi-total point graph for the same.

Keywords: K.Banhatti indices; Homeomorphism; Graph operator.

1. Introduction

Topological indices are useful tools to model physical and chemical properties of molecules to design pharmacologically active compounds, to recognize environmentally hazardous materials etc., [1]. Let $G(V, E)$ be a connected graph with $|V(G)| = n$ vertices and $|E(G)| = m$ edges. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge $e = uv$ in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. The vertices and edges of a graph are said to be its elements [2].

A chemical graph is a graph in which the vertices correspond to the atoms and the edges to the bonds of a chemical compound. A single number obtained from the chemical graph is that to characterize some property of the underlying chemical is said to be a topological index or molecular structure descriptor. A lot of such descriptors have been considered in theoretical chemistry and have some applications especially in QSPR/QSAR fields of research (see [6, 8, 10, 12]).

The K.Banhatti indices play a very important role in the investigation of chemical properties of a chemical compound. This paper discusses the K.Banhatti indices of some special graphs as well as a vertex gluing of web graph by finding general formula and also for subdivision and semi total point graph for same graph.

2. Basic Definitions

Definition 2.1. *The first and second Banhatti indices were first introduced by Kulli* [2, 3] *and are defined as follows:*

$$B_1(G) = \sum_{ue}[d_G(u) + d_G(e)] \text{ and } B_2(G) = \sum_{ue} d_G(u).d_G(e)$$

where ue means that the vertex u and edge e are incident in G.

Definition 2.2. *The k Hyper-Banhatti indices were defined by Kulli [5] and are as below.*

$$HB_1(G) = \sum_{ue}[d_G(u) + d_G(e)]^2 \text{ and } HB_2(G) = \sum_{ue}[d_G(u).d_G(e)]^2$$

where ue means that the vertex u and edge e are incident in G.

Definition 2.3. [11] *The subdivision graph $S(G)$ of a graph η is the graph obtained by adding a new vertex of degree 2 in each edge of G.*

Definition 2.4. [7] *The semi-total line graph $R(G)$ is the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it.*

A K_4-homeomorphic graph or simply K_4-homeomorph, denoted by $K_4(e_1, e_2, e_3, e_4, e_5, e_6)$ is the graph obtained when the six edges of a complete graph with four vertices (K_4) are subdivided edge is called a path and its length is the number of resulting segments (see Fig. 1). A complete bipartite graph is

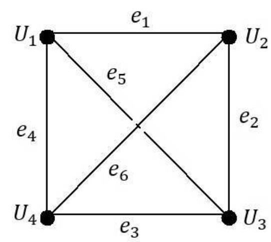

Fig. 1. K_4 - Homeomorphic graph

a simple bipartite graph with partite sets U_1 and U_2, where every vertex in U_1 is adjacent with all the vertices in U_2. If $|U_1| = m$ and $|U_2| = n$, then such complete bipartite graph is denoted by $K_{m,n}$ [or $K(m, n)$]. So $K_{m,n}$ has order $m+n$ and size mn (see Fig. 2). A graph consisting of r paths joining two vertices is

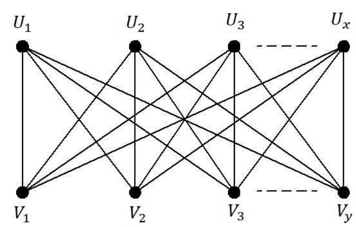

Fig. 2. A complete bipartite graph $K_{x,y}$

called an r-bridge graph, which is denoted by $T(e_1, e_2 \cdots e_r)$, where $e_1, e_2 \cdots e_r$ are the lengths of r paths. Clearly, an r-bridge graph is a generalized polygon tree (see Fig. 3).

A web graph $Web(r, s)$ is the graph obtained from the cartesian product of the cycle C_r and the path P_s (see Fig. 4).

3. The K.Banhatti Indices of Some Special Graphs

This section demonstrates general formulas obtained for some special graphs.

Theorem 3.1. *Let* $e_1, e_2, e_3, e_4, e_5, e_6$ *be positive integers. Then the K.Banhatti*

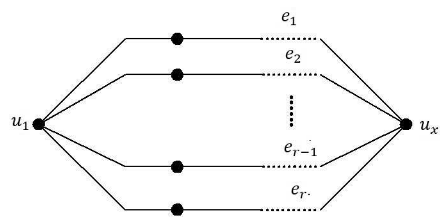

Fig. 3. r-bridge graph

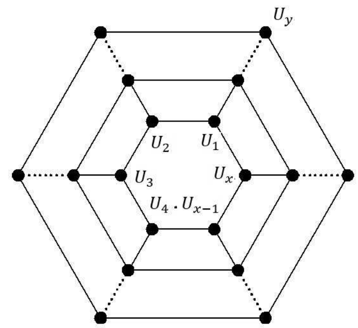

Fig. 4. A web graph $Web(x, y)$

indices of a K_4-homeomorphism graph denoted by $K_4(e_1, e_2, e_3, e_4, e_5, e_6)$ will be as follows:

(i) *If e_1 or/and e_2 or/and e_3 or/and e_4 or/and e_5 or/and $e_6 = 1$, then the first and second Banhatti indices to any one of them is, 14 and 24 respectively.*

(ii) *If e_1 or/and e_2 or/and e_3 or/and e_4 or/and e_5 or/and $e_6 \neq 1$ then the first and second Banhatti indices to any one of them is, (number of edges) 11 and (number of edges) 15 respectively.*

Proof. (i) If e_1 or/and e_2 or/and e_3 or/and e_4 or/and e_5 or/and $e_6 = 1$, then any one of them will have one edge and two vertices with the same degree three. Thus the first and second Banhatti indices to any one of them is 14 and 24 respectively.

(ii) If e_1 or/and e_2 or/and e_3 or/and e_4 or/and e_5 or/and e_6, then any one of them will have two or more edges and each of them will have two vertices in which at least one of the vertices is of degree two. Thus, the first and second Banhatti indices to any one of them is (number of edges) 11 and (number of edges) 15, respectively. ∎

Theorem 3.2. *Let m, n be positive integers. The first and second Banhatti indices* *of a complete bipartite graph denoted by $K_{m,n}$ are,*

$$B_1[K_{m,n}] = mn[3m+3n-4]$$
$$B_2[K_{m,n}] = mn(m+n)(m+n-2).$$

Proof. In complete bipartite graph, we have mn number of edges each one of them has two vertices that have same degree which has the first vertex of degree m and the second vertex of degree n. Hence by the definitions of first and second Banhatti indices, we get,

$$B_1[K_{m,n}] = mn[3m+3n-4]$$
$$B_2[K_{m,n}] = mn(m+n)(m+n-2).$$

■

Theorem 3.3. *Let k be a positive integer. The first and second Banhatti indices of a k-bridge graph denoted by $T(e_1, e_2, , , , , , e_k)$ are,*

$$B_1[T(e_1, e_2, , , , , , e_k)] = (e_1 + e_2 +, , , , e_k)8$$
$$B_2[T(e_1, e_2, , , , , , e_k)] = (e_1 + e_2 +, , , , e_k)8.$$

Proof. The above result is proved by mathematical induction.

Let $K = 2$. Then $G = T(e_1, e_2)$, whose graph as follows: Thus,

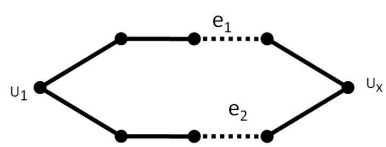

Fig. 5. 2−bridge graph

$$B_1[T(e_1, e_2)] = (e_1 + e_2)[3(2) + 3(2) - 4] = (e_1 + e_2)8,$$
$$B_2[T(e_1, e_2)] = (e_1 + e_2)[(2+2)^2 - 2(2+2)] = (e_1 + e_2)8.$$

Hence, it is true for $k = 2$.

Induction step: Assume that the result is true for $k = r$.

$$B_1[T(e_1, e_2 \cdots e_r)] = (e_1 + e_2 + \cdots + e_r)8$$
$$B_2[T(e_1, e_2 + \cdots + e_r)] = (e_1 + e_2 + \cdots + e_r)8.$$

Now, we have to prove that the result is true for $k = r + 1$.

Consider a graph with $r+1$ bridges. where e_i denotes the position of the edges of graph $T(e_1, e_2, , , , , e_r)$ at the i^{th} position. The graph H is the path which contains endings V_1 and V_2 and e_{r+1} is the number of edges in H as follows.

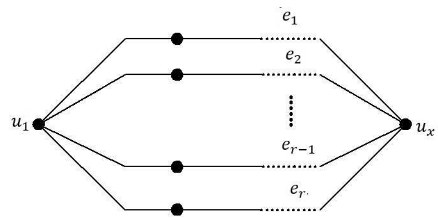

Fig. 6. $r-$bridge

Fig. 7. $r+1$ path graph

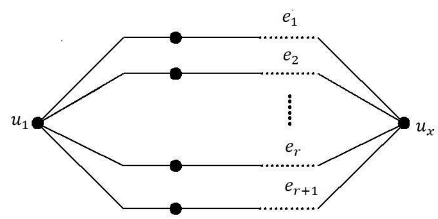

Fig. 8. $r+1-$bridge graph

Connect the graph $T(e_1, e_2, , , , , , e_r)$ with the graph H such that $V_1 = U_1$ and $V_2 = U_2$. The vertices $V_1 = U_1$ and $V_2 = U_2$ are of degree $r+1$, as follows: Thus,

$$\begin{aligned} B_1(T_{r+1}) &= B_1(T_r) + B_1(H) \\ &= 8(e_1 + e_2 + e_3 + e_4 + e_5 + e_6) + 8e_{r+1} \\ &= (e_1 + e_2 + \cdots + e_{r+1})8. \\ B_2(T_{r+1}) &= B_2(T_r) + B_2(H) \\ &= 8(e_1 + e_2 + e_3 + e_4 + e_5 + e_6) + 8e_{r+1} \\ &= (e_1 + e_2 + \cdots + 8e_{r+1}). \end{aligned}$$

Hence, the result is true for $k = r+1$.

By induction, the result is true for all k.

$$B_1(T_{r+1}) = (e_1 + e + 2+, , , , e_r)8 = B_2(T_{r+1}) \qquad \blacksquare$$

4. The K.Banhatti Indices of Vertex Gluing Web Graph

This section contains the general formulas obtained for K.Banhatti indices and

k-hyper Banhatti indices of vertex gluing web graph and subdivision and semi-total point graph for the same.

Let u_1-gluing of web graph be a graph obtained from two different web graphs Web(x,p) and Web(y,q) with one common vertex u_1 denoted by $W^{y,q}_{x,p}(u_1)$ (vertex gluing of graph) (see Fig. 9).

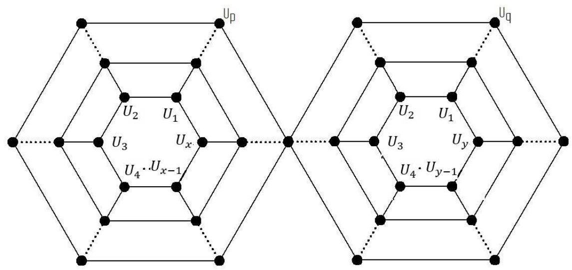

Fig. 9. U_1- gluing of Web Graph $W^{y,q}_{x,p}(u_1)$

Theorem 4.1. *Let x,p,y and q be positive integers. Then the first and second Banhatti indices of the u_1- gluing of web graph $W^{y,q}_{x,p}(u_1)$ are*

$$B_1[W^{y,q}_{x,p}(u_1)] = \begin{cases} 52(x+y-2)+138 & \text{if } p,q=2 \\ 14(3x+2y-5)+17(2y-1)+20y(2q-5)+141 & \text{if } p=2 \\ 28(x+y-2)+34(x+y-1)+20[x(2p-5) + y(2q-5)]+144 & \text{if } p,q\neq 2. \end{cases} \quad (1)$$

$$B_2[W^{y,q}_{x,p}(u_1)] = \begin{cases} 72(x+y-2)+378 & \text{if } p,q=2, \\ 24(3x+2y-5)+35(2y-1)+48y(2q-5)+395 & \text{if } p=2, \\ 48(x+y-2)+70(x+y-1)+48[x(2p-5) + y(2q-5)]+412 & \text{if } p,q\neq 2. \end{cases}$$

Proof. We have three cases and their edge and vertex partitions of above web graph are as follows:

Case (i):

(3,3)	(3,6)
$3(x+y-2)$	6

By definitions of K.Banhatti indices, we get

$$B_1[W^{y,q}_{x,p}(u_1)] = 52(x+y-2)+138,$$
$$B_2[W^{y,q}_{x,p}(u_1)] = 72(x+y-2)+378.$$

Case (ii):

(3,3)	(3,4)	(3,6)	(4,4)	(4,6)
$(3x+2y-5)$	$(2y-1)$	5	$y(2q-5)$	1

By definitions of K.Banhatti indices, we get

$$B_1[W_{x,p}^{y;q}(u_1)] = 14(3x+2y-5)+17(2y-1)+20y(2q-5)+141,$$
$$B_2[W_{x,p}^{y;q}(u_1)] = 24(3x+2y-5)+35(2y-1)+48y(2q-5)+395.$$

Case (iii):

(3,3)	(3,4)	(3,6)	(4,4)	(4,6)
$2(x+y-2)$	$2(x+y-1)$	4	$x(2p-5)+y(2q-5)$	2

By definitions of K.Banhatti indices, we get

$$\begin{aligned} &B_1[W_{x,p}^{y;q}(u_1)] \\ =\ &28(x+y-2)+34(x+y-1)+20[x(2p-5)+y(2q-5)]+144, \\ &B_2[W_{x,p}^{y;q}(u_1)] \\ =\ &48(x+y-2)+70(x+y-1)+48[x(2p-5)+y(2q-5)]+412. \end{aligned}$$

Hence by combining above three cases, we get

$$B_1[W_{x,p}^{y;q}(u_1)]$$
$$= \begin{cases} 52(x+y-2)+138 & \text{if } p,q=2, \\ 14(3x+2y-5)+17(2y-1)+20y(2q-5)+141 & \text{if } p=2, \\ 28(x+y-2)+34(x+y-1)+20[x(2p-5) \\ \quad +y(2q-5)]+144 & \text{if } p,q\neq 2, \end{cases}$$

$$B_2[W_{x,p}^{y;q}(u_1)]$$
$$= \begin{cases} 72(x+y-2)+378 & \text{if } p,q=2, \\ 24(3x+2y-5)+35(2y-1)+48y(2q-5)+395 & \text{if } p=2, \\ 48(x+y-2)+70(x+y-1)+48[x(2p-5) \\ \quad +y(2q-5)]+412 & \text{if } p,q\neq 2. \end{cases}$$

■

Theorem 4.2. *Let x, p, y and q be positive integers. Then the subdivision of first and second Banhatti indices of the u_1- gluing of web graph $W_{x,p}^{y;q}(u_1)$ are*

$$S[B_1[W_{x,p}^{y;q}(u_1)]] = \begin{cases} 66(x+y+1)-12 & \text{if } p,q=2 \\ 6(11x+9)+2y(28q-23) & \text{if } p=2 \\ 28q(x-y)-2(23x-33y+27) & \text{if } p,q\neq 2. \end{cases} \tag{2}$$

$$S[B_2[W_{x,p}^{y;q}(u_1)]] = \begin{cases} 90(x+y-1)+288 & \text{if } p,q=2 \\ 2(45x+99)+2y(48q-51) & \text{if } p=2 \\ 48q(x-y)-2(51x-45y+99) & \text{if } p,q\neq 2. \end{cases} \tag{3}$$

Proof. We have three cases and their edge and vertex partitions of the above web graph are as follows:

Case (i):

(2,3)	(2,6)
$6(x+y-1)$	6

By definitions of K.Banhatti indices, we get

$$\begin{aligned} S[B_1[W^{y,q}_{x,p}(u_1)]] &= 66(x+y+1)-12, \\ S[B_2[W^{y,q}_{x,p}(u_1)]] &= 90(x+y-1)+288. \end{aligned}$$

Case (ii):

(2,3)	(2,4)	(2,6)
$6(x+y-1)$	$4y(q-2)$	6

By definitions of K.Banhatti indices, we get

$$\begin{aligned} S[B_1[W^{y,q}_{x,p}(u_1)]] &= 6(11x+9)+2y(28q-23), \\ S[B_2[W^{y,q}_{x,p}(u_1)]] &= 2(45x+99)+2y(48q-51). \end{aligned}$$

Case (iii):

(2,3)	(2,4)	(2,6)
$6(x+y-1)$	$2q(x-y)-8x$	6

By definitions of K.Banhatti indices, we get

$$\begin{aligned} S[B_1[W^{y,q}_{x,p}(u_1)]] &= 28q(x-y)-2(23x-33y+27), \\ S[B_2[W^{y,q}_{x,p}(u_1)]] &= 48q(x-y)-2(51x-45y+99). \end{aligned}$$

Hence by combining above three cases, we get

$$S[B_1[W^{y,q}_{x,p}(u_1)]] = \begin{cases} 66(x+y+1)-12 & \text{if } p,q=2 \\ 6(11x+9)+2y(28q-23) & \text{if } p=2 \\ 28q(x-y)-2(23x-33y+27) & \text{if } p,q\neq 2. \end{cases}$$

$$S[B_2[W^{y,q}_{x,p}(u_1)]] = \begin{cases} 90(x+y-1)+288 & \text{if } p,q=2 \\ 2(45x+99)+2y(48q-51) & \text{if } p=2 \\ 48q(x-y)-2(51x-45y+99) & \text{if } p,q\neq 2. \end{cases}$$ ∎

Theorem 4.3. *Let x,p,y and q be positive integers. Then the semi total point graph of first and second Banhatti indices of the u_1- gluing of web graph $W^{y,q}_{x,p}(u_1)$ are*

$$R[B_1[W^{y,q}_{x,p}(u_1)]] = \begin{cases} 216(x+y+1) & \textit{if } p,q=2 \\ 72(3x+1)+8y(24q-21)+8 & \textit{if } p=2 \\ 4(y-2x-11)-16x(q+10)-12y(7q+1) & \textit{if } p,q\neq 2. \end{cases} \quad (4)$$

$$R[B_2[W_{x,p}^{y,q}(u_1)]] = \begin{cases} 648(x+y+2)+432 & \textit{if } p,q=2 \\ 588(x+3)+4y(192q-239)+68 & \textit{if } p=2 \\ 24x(32q-37)+32y(14q-13)-8(x-112) & \textit{if } p,q\neq 2. \end{cases} \quad (5)$$

Proof. We have three Cases and their edge and vertex partition of above web graph as follow.

Case (i):

(2,6)	(2,12)	(6,6)	(6,12)
$6(x+y-1)$	6	$3(x+y-2)$	6

Then by definitions of K.Banhatti indices, we get

$$R[B_1[W_{x,p}^{y,q}(u_1)]] = 216(x+y+1),$$
$$R[B_2[W_{x,p}^{y,q}(u_1)]] = 648(x+y+2)+432.$$

Case (ii):

(2,3)	(2,8)	(2,12)	(6,6)	(6,8)	(6,12)	(8,8)	(8,12)
$6(x+y-1)$	$4y(q-2)$	6	$3x+2y-5$	$2y-1$	5	$y(2q-5)$	1

Then by definitions of K.Banhatti indices, we get

$$R[B_1[W_{x,p}^{y,q}(u_1)]] = 72(3x+1)+8y(24q-21)+8,$$
$$R[B_2[W_{x,p}^{y,q}(u_1)]] = 588(x+3)+4y(192q-239)+68.$$

Case (iii):

(2,6)	(2,8)	(2,12)	(6,6)	(6,8)	(6,12)	(8,8)	(8,12)
$6(x+y-1)$	$4q(x-y)-2(4x-y+5)$	6	$2(x+y-2)$	$2(x+y-1)$	4	$(x+y)(2q-5)$	2

Then by definitions of K.Banhatti indices, we get

$$R[B_1[W_{x,p}^{y,q}(u_1)]] = 4(y-2x-11)-16x(q+10)-12y(7q+1),$$
$$R[B_2[W_{x,p}^{y,q}(u_1)]] = 24x(32q-37)+32y(14q-13)-8(x-112).$$

Hence by combining above three cases, we get

$$R[B_1[W_{x,p}^{y,q}(u_1)]] = \begin{cases} 216(x+y+1) & \text{if } p,q=2, \\ 72(3x+1)+8y(24q-21)+8 & \text{if } p=2, \\ 4(y-2x-11)-16x(q+10)-12y(7q+1) & \text{if } p,q\neq 2, \end{cases} \quad (6)$$

$$R[B_2[W_{x,p}^{y;q}(u_1)]] = \begin{cases} 648(x+y+2)+432 & \text{if } p,q=2, \\ 588(x+3)+4y(192q-239)+68 & \text{if } p=2, \\ 24x(32q-37)+32y(14q-13)-8(x-112) & \text{if } p,q \neq 2. \end{cases} \quad (7)$$

■

Theorem 4.4. *Let x, p, y and q be positive integers. Then the subdivision of first and second k-hyper Banhatti indices of the u_1- gluing of web graph $W_{x,p}^{y;q}(u_1)$ are*

$$S[HB_1[W_{x,p}^{y;q}(u_1)]] = \begin{cases} 726(x+y+2)+222 & \textit{if } p,q=2, \\ 726(x-y+2)+116y(6q-1)-2(44yq+111) & \textit{if } p=2, \\ 726(x+y)+784q(x-y)-2(784x-837) & \textit{if } p,q \neq 2, \end{cases} \quad (8)$$

$$S[HB_2[W_{x,p}^{y;q}(u_1)]] = \begin{cases} 1350(x+y+9)+324 & \textit{if } p,q=2, \\ 1350(x-2y+9)+558y(4q-1)+36(2yq-9) & \textit{if } p=2, \\ 1350(x+y)+2304q(x-y)-18(256x-693) & \textit{if } p,q \neq 2. \end{cases} \quad (9)$$

Proof. We have three Cases and their edge and vertex partition of above web graph as follow.

Case (i):

(2,3)	(2,6)
$6(x+y-1)$	6

Then by definitions of K-Hyper Banhatti indices, we get

$$S[HB_1[W_{x,p}^{y;q}(u_1)]] = 726(x+y+2)+222,$$
$$S[HB_2[W_{x,p}^{y;q}(u_1)]] = 1350(x+y+9)+324.$$

Case (ii):

(2,3)	(2,4)	(2,6)
$6(x+y-1)$	$4y(q-2)$	6

Then by definitions of K-Hyper Banhatti indices, we get

$$S[HB_1[W_{x,p}^{y;q}(u_1)]] = 726(x-y+2)+116y(6q-1)-2(44yq+111),$$
$$S[HB_2[W_{x,p}^{y;q}(u_1)]] = 1350(x-2y+9)+558y(4q-1)+36(2yq-9).$$

Case (iii):

(2,3)	(2,4)	(2,6)
$6(x+y-1)$	$2q(x-y)-8x$	6

Then by definitions of K-Hyper Banhatti indices, we get

$$\begin{aligned} S[HB_1[W^{y,q}_{x,p}(u_1)]] &= 726(x+y)+784q(x-y)-2(784x-837),\\ S[HB_2[W^{y,q}_{x,p}(u_1)]] &= 1350(x+y)+2304q(x-y)-18(256x-693). \end{aligned}$$

Hence by combining above three cases, we get

$$\begin{aligned} &S[HB_1[W^{y,q}_{x,p}(u_1)]]\\ &=\begin{cases} 726(x+y+2)+222 & \text{if } p,q=2\\ 726(x-y+2)+116y(6q-1)-2(44yq+111) & \text{if } p=2\\ 726(x+y)+784q(x-y)-2(784x-837) & \text{if } p,q\neq 2. \end{cases} \end{aligned}$$

$$\begin{aligned} &S[HB_2[W^{y,q}_{x,p}(u_1)]]\\ &=\begin{cases} 1350(x+y+9)+324 & \text{if } p,q=2\\ 1350(x-2y+9)+558y(4q-1)+36(2yq-9) & \text{if } p=2\\ 1350(x+y)+2304q(x-y)-18(256x-693) & \text{if } p,q\neq 2. \end{cases} \end{aligned}$$ ■

Theorem 4.5. *Let x,p,y and q be positive integers. Then the semi total point graph of first and second k-hyper Banhatti indices of the u_1- gluing of web graph $W^{y,q}_{x,p}(u_1)$ are*

$$\begin{aligned} &R[HB_1[W^{y,q}_{x,p}(u_1)]]\\ &=\begin{cases} 5476(x+y+2)+4176 & \textit{if } p,q=2\\ 5472(x-y+2)+24y(274q-95)+3756 & \textit{if } p=2\\ 8(446y-399x+1099)+16q(297x-41y) & \textit{if } p,q\neq 2. \end{cases} \end{aligned} \tag{10}$$

$$\begin{aligned} &R[HB_2[W^{y,q}_{x,p}(u_1)]]\\ &=\begin{cases} 57024(x+y+7)+15206 & \textit{if } p,q=2\\ 57024(x-3y+10)+64y(1968q-499)-29376 & \textit{if } p=2\\ 1024q(123x+73y)-256(793x+543y)+568448 & \textit{if } p,q\neq 2. \end{cases} \end{aligned} \tag{11}$$

Proof. We have three Cases and their edge and vertex partition of above web graph as follow.

Case (i):

(2,6)	(2,12)	(6,6)	(6,12)
$6(x+y-1)$	6	$3(x+y-2)$	6

Then by definitions of K.Banhatti indices, we get

$$\begin{aligned} R[HB_1[W^{y,q}_{x,p}(u_1)]] &= 5476(x+y+2)+4176,\\ R[HB_2[W^{y,q}_{x,p}(u_1)]] &= 57024(x+y+7)+15206. \end{aligned}$$

Case (ii):

(2,3)	(2,8)	(2,12)	(6,6)	(6,8)	(6,12)	(8,8)	(8,12)
$6(x+y-1)$	$4y(q-2)$	6	$3x+2y-5$	$2y-1$	5	$y(2q-5)$	1

Then by definitions of K.Banhatti indices, we get

$$R[HB_1[W_{x,p}^{y,q}(u_1)]] = 5472(x-y+2)+24y(274q-95)+3756,$$
$$R[HB_2[W_{x,p}^{y,q}(u_1)]] = 57024(x-3y+10)+64y(1968q-499)-29376.$$

Case (iii):

(2,6)	(2,8)	(2,12)	(6,6)	(6,8)	(6,12)	(8,8)	(8,12)
$6(x+y-1)$	$4q(x-y)-2(4x-y+5)$	6	$2(x+y-2)$	$2(x+y-1)$	4	$(x+y)(2q-5)$	2

Then by definitions of K.Banhatti indices, we get

$$R[HB_1[W_{x,p}^{y,q}(u_1)]] = 8(446y-399x+1099)+16q(297x-41y),$$
$$R[HB_2[W_{x,p}^{y,q}(u_1)]] = 1024q(123x+73y)-256(793x+543y)+568448.$$

Hence by combining above three cases, we get

$$R[HB_1[W_{x,p}^{y,q}(u_1)]] = \begin{cases} 5476(x+y+2)+4176 & \text{if } p,q=2 \\ 5472(x-y+2)+24y(274q-95)+3756 & \text{if } p=2 \\ 8(446y-399x+1099)+16q(297x-41y) & \text{if } p,q\neq 2. \end{cases}$$

$$R[HB_2[W_{x,p}^{y,q}(u_1)]] = \begin{cases} 57024(x+y+7)+15206 & \text{if } p,q=2 \\ 57024(x-3y+10)+64y(1968q-499)-29376 & \text{if } p=2 \\ 1024q(123x+73y)-256(793x+543y)+568448 & \text{if } p,q\neq 2. \end{cases} \qquad \blacksquare$$

5. Conclusions

The paper has introduced the general formula for K.Banhatti indices of certain graphs namely K_4-homeomorphism, complete bipartite, k-bridge graphs and vertex gluing of web graphs and also subdivision and semi total graph for K.Banhatti indices and K-Hyper Banhatti indices for the same.

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