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Ideal as a Catalyst to Nano Topology

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Abstract. This article presents a new formulation of nano toplogical structure for a given ideal. The choice of the ideal has its own impact on the nano approximations. Also we discuss some of their properties and a comparative analysis have been done based on nano approximations..The nano approximations via ideal results in diminished upper approximation, enlarged lower approximation and elevated accuracy.

Keywords: Nano topology; Binary reflexive relation; Nano Accuracy; Nano measure.

1. Introduction

The concept of ideal in topological space was first introduced by Kuratowski [6]. Lellis Thivagar et al. [7] interjected a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. The topology recommended here is named so because of its size, since it has atmost five elements in it. In this paper new definitions of lower and upper approximations via ideal have been introduced based on both any binary reflexive relation[1] and equivalence relation [7]. A comparative analysis have been done with nano approximations. It is therefore shown that the current definitions are more general. It is apparent that the present method also decreases the boundary region and we get a topology finer than existing one. In fact, the imprecision of a set is caused by its boundary region. If the boundary region of a set is larger then imprecision is larger. The process of analyzing data under uncertainty is the main goal for many real life problems. In the present paper, we have made comparisions based on Nano measure.

2. Preliminaries

The following recalls necessary concepts and preliminaries required in the sequel of our work.

Definition 2.1. [7] *Let $\mathcal{U}$ be a non-empty finite set of objects called the universe $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.*

(i) *The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{\bigcup_{x\in\mathcal{U}}\{R(x) : R(x) \subseteq X\}\right\}$, where R(x) denotes the equivalence class determined by x.*

(ii) *The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \left\{\bigcup_{X\in\mathcal{U}}\{R(x) : R(x) \cap X \neq \emptyset\}\right\}$.*

(iii) *The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. [7] *Let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:*

(i) *$\mathcal{U}$ and $\emptyset \in \tau_R(X)$*

(ii) *The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

(iii) *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ forms a topology on $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. We call $\{\mathcal{U}, \tau_R(X)\}$ as the nano topological space.

Remark 2.3. Similarly, the above definition valids if R is reflexive relation.

Definition 2.4. [2] *An ideal $\mathcal{I}$ on a topological space is a non-empty collection of subsets of X which satisfies*

(i) $A \in \mathcal{I}$ *and* $B \subset A \Rightarrow B \in \mathcal{I}$.
(ii) $A \in \mathcal{I}$ *and* $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$.

Definition 2.5. [1] *If* R *is any binary relation on* $\mathcal{U}$*, then the aftersets of* $x \in \mathcal{U}$ *is* $xR = \{y : xRy\}$.

3. Nano Ideal Topology Based on Reflexive Relation

In classical, Ideal topology is a topological space together with an ideal, but in the Nano ideal topological structure ideal plays an vital role in the nano approximations.

Definition 3.1. *Let* R *be a reflexive binary relation on* $\mathcal{U}$*. For any set* $X \subseteq \mathcal{U}$*, a pair of lower and upper approximation are denoted by* $\underline{R}(X)$ *and* $\overline{R}(X)$

(i) $\underline{R}(X) = \{x \in \mathcal{U} :< x > R \subseteq X\}$
(ii) $\overline{R}(X) = \{x \in \mathcal{U} :< x > R \cap X \neq \emptyset\}$
(iii) $Bnd(X) = \overline{R}(X) - \underline{R}(X)$

Then $\tau_R(X) = \{\mathcal{U}, \emptyset, \underline{R}(X), \overline{R}(X), Bnd(X)\}$ *forms a topology on* $\mathcal{U}$*. We call* $\{\mathcal{U}, \tau_R(X)\}$ *as the nano topological space based on reflexive relation.*

Definition 3.2. *Let* R *be a reflexive binary relation on* $\mathcal{U}$*, and* $\mathcal{I}$ *be an ideal on* $\mathcal{U}$*. For any set* $X \subseteq \mathcal{U}$*, the lower and upper approximation* $\underline{R}_{\mathcal{I}}(X)$*,* $\overline{R}_{\mathcal{I}}(X)$*are defined by*

(i) $\underline{R}_{\mathcal{I}}(X) = \{x \in \mathcal{U} :< x >_R \cap X^c \in \mathcal{I}\}$
(ii) $\overline{R}_{\mathcal{I}}(X) = X \cup \{x \in \mathcal{U} :< x >_R \cap X \notin \mathcal{I}\}$
(iii) $Bnd_{\mathcal{I}}(X) = \overline{R}_{\mathcal{I}}(X) - \underline{R}_{\mathcal{I}}(X)$

where,$< p >_R = \cap\{xR$ *if there exist* $x : p \in xR\}$ *is the intersection of all aftersets containing* p.

Then $\tau_{R_{\mathcal{I}}}(X) = \{\mathcal{U}, \emptyset, \overline{R}_{\mathcal{I}}(X), \underline{R}_{\mathcal{I}}(X), Bnd_{\mathcal{I}}(X)\}$ *forms a topology on* $\mathcal{U}$ *called as the R-nano ideal topology on* $\mathcal{U}$ *with respect to X. We call* $(\mathcal{U}, \tau_{R_{\mathcal{I}}}(X))$ *as the R-nano ideal topological space.*

Example 3.3. Let $\mathcal{U} = \{a, b, c, d\}$, $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, c), (c, d), (b, d)\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$, where $< a >_R = \{a, b, c\}$,$< b >_R = \{b\}$, $< c >_R = \{c\}$, $< d >_R = \{d\}$ and let $X = \{b, c\} \subseteq \mathcal{U}$. Then $\overline{R}_{\mathcal{I}}(X) = \{b, c\}$, $\underline{R}_{\mathcal{I}}(X) = \{b, c\}$ and $Bnd_{\mathcal{I}}(X)\} = \emptyset$. Therefore the R- nano topology induced by an ideal $\tau_{R_{\mathcal{I}(X)}}) = \{\mathcal{U}, \emptyset, \{b, c\}\}$.

Theorem 3.4. *Let* R *be a reflexive binary relation on* $\mathcal{U}$ *and Let* $\mathcal{I}$ *and* $\mathcal{J}$ *be ideals on* $\mathcal{U}$ *and also* $X, Y \subseteq \mathcal{U}$*. Then we have*

(i) $\overline{R}_{\mathcal{I}}(\emptyset) = \emptyset$

(ii) $X \subseteq \overline{R}_{\mathcal{I}}(X)$
(iii) $X \subseteq Y \Rightarrow \overline{R}_{\mathcal{I}}(X) \subseteq \overline{R}_{\mathcal{I}}(Y)$
(iv) $\overline{R}_{\mathcal{I}}(X \cup Y) = \overline{R}_{\mathcal{I}}(X) \cup \overline{R}_{\mathcal{I}}(Y)$
(v) $\overline{R}_{\mathcal{I}}(X) = [\underline{R}_{\mathcal{I}}(X^c)]^c$

Proof. (i) Obvious.

(ii) X $\subseteq \overline{R}_{\mathcal{I}}(X)$, which follows clearly from the definition.

(iii) Given X $\subseteq Y$ and also X $\subseteq \overline{R}_{\mathcal{I}}(X)$ and Y $\subseteq \overline{R}_{\mathcal{I}}(Y)$. Let $x \in \overline{R}_{\mathcal{I}}(X)$ and since $X \subseteq \overline{R}_{\mathcal{I}}(X)$ and $X \subseteq Y$ also $x \in X \Rightarrow x \in Y \Rightarrow x \in \overline{R}_{\mathcal{I}}(Y)$.

(iv) Let $x \in \overline{R}_{\mathcal{I}}(X \cup Y)$, implies that $< x >_R \cap (X \cup Y) \notin \mathcal{I}$. If $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$, then $\overline{R}_{\mathcal{I}}(X) \subseteq \overline{R}_{\mathcal{I}}(X \cup Y)$ and $\overline{R}_{\mathcal{I}}(Y) \subseteq \overline{R}_{\mathcal{I}}(X \cup Y)$ implies that $\overline{R}_{\mathcal{I}}(X) \cup \overline{R}_{\mathcal{I}}(Y) \subseteq \overline{R}_{\mathcal{I}}(X \cup Y)$. Further $X \subseteq \overline{R}_{\mathcal{I}}(X)$ and $Y \subseteq \overline{R}_{\mathcal{I}}(Y)$ also $X \cup Y \subseteq \overline{R}_{\mathcal{I}}(X) \cup \overline{R}_{\mathcal{I}}(Y)$. But $X \cup Y \subseteq \overline{R}_{\mathcal{I}}(X \cup Y) \subseteq \overline{R}_{\mathcal{I}}(X) \subseteq \overline{R}_{\mathcal{I}}(Y)$. Therefore $\overline{R}_{\mathcal{I}}(X \cup Y) = \overline{R}_{\mathcal{I}}(X) \cup \overline{R}_{\mathcal{I}}(Y)$.

(v) $[\underline{R}_{\mathcal{I}}(X^c)]^c = \{\{x \in X^c :< x >_R \cap X^c \in \mathcal{I}\}\}^c = X \cup \{x \in \mathcal{U} :< x >_R \cap X \notin \mathcal{I}\} = \overline{R}_{\mathcal{I}}(X)$. ∎

4. Comparision Triggered by Approximations

In this section we compare nano topological space and nano ideal topology based on reflexive relation.

Remark 4.1. Most researches for the development of theory are directed to increase certainty of approximation by contraction of boundary region. In fact, the imprecision of a set is caused by its boundary region. Larger the boundary region of a set then the imprecision also increases. Imprecision in this approach is expressed by boundary region of a set. Instead of nano topology, nano ideal topology based on reflexive relation leads us to get to a mechanism for decreasing the boundary regions and making it as small as possible

Example 4.2. Let $\mathcal{U} = \{a, b, c, d\}$, R $= \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, d), (b, c), (c, b)\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ be an ideal on $\mathcal{U}$

Remark 4.3. The following theorem reveals that the choice of larger ideal leads to smaller boundary.

Theorem 4.4. *Let R be a reflexive binary relation on $\mathcal{U}$ and $\mathcal{I}$ and $\mathcal{J}$ be two ideals on $\mathcal{U}$. If $\mathcal{I} \subseteq \mathcal{J}$, then $Bnd_J(A) \subseteq Bnd_I(A)$.*

Proof. Let $x \in Bnd_J(A)$. Then $x \in \overline{R}_{\mathcal{J}(A)}$ and $x \in (\underline{R}_{\mathcal{J}(A)})^c$. It follows that $x \in \overline{R}_{\mathcal{I}(A)}$ and $x \in (\underline{R}_{\mathcal{I}(A)})^c$. Hence $x \in Bnd_{\mathcal{I}}(A)$. ∎

Remark 4.5. The following example is the consequence of the above theorem.

X	$\underline{R}(X)$	$\underline{R}_{\mathcal{I}}(X)$	$\overline{R}(X)$	$\overline{R}_{\mathcal{I}}(X)$	$Bnd(X)$	$Bnd_{\mathcal{I}}(X)$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\emptyset$	$\emptyset$
$\{a\}$	$\emptyset$	$\emptyset$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
$\{b\}$	$\{b\}$	$\{b\}$	$\{a,b,c\}$	$\{b\}$	$\{a,c\}$	$\emptyset$
$\{c\}$	$\emptyset$	$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$	$\emptyset$
$\{d\}$	$\{d\}$	$\{d\}$	$\{a,d\}$	$\{a,d\}$	$\{a\}$	$\{a\}$
$\{a,b\}$	$\{b\}$	$\{b\}$	$\{a,b,c\}$	$\{a,b\}$	$\{a,c\}$	$\emptyset$
$\{a,c\}$	$\emptyset$	$\{c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a\}$
$\{a,d\}$	$\{d\}$	$\{a,d\}$	$\{a,d\}$	$\{a,d\}$	$\{a\}$	$\emptyset$
$\{b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{b,c\}$	$\{a\}$	$\emptyset$
$\{b,d\}$	$\{b,d\}$	$\{b,d\}$	$\mathcal{U}$	$\{a,b,d\}$	$\{a,c\}$	$\{a\}$
$\{c,d\}$	$\{d\}$	$\{c,d\}$	$\{c,d\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a\}$
$\{a,b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a\}$	$\{a\}$
$\{a,b,d\}$	$\{a,b,d\}$	$\{a,b,d\}$	$\mathcal{U}$	$\{a,b,d\}$	$\{c\}$	$\emptyset$
$\{a,c,d\}$	$\{d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c\}$	$\emptyset$
$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\mathcal{U}$	$\mathcal{U}$	$\{a\}$	$\{a\}$

Table 1

Example 4.6. Let $\mathcal{U} = \{a,b,c,d\}$, $R = \{(a,a),(b,b),(c,c),(d,d),(a,b),(a,d),(b,c),(c,b)\}$, $X = \{b\} \subseteq \mathcal{U}$. Let $\mathcal{I} = \{\emptyset, \{b\}\}$ and $\mathcal{J} = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$ where $Bnd_J(A) \subseteq Bnd_I(A)$. Then $Bnd_{\mathcal{J}}(X) = \emptyset \subseteq \{a,c\} = Bnd_{\mathcal{I}}(X)$.

5. Analogy in Terms of Measure

Here, we consider a multi-valued information table containing the records of applicants for a job and using the data nano topology and nano ideal topology based on reflexive relation were found and paralleled in terms of nano accuracy.

Definition 5.1. *Let $(\mathcal{U}, A)$ be an information system where $\mathcal{U}$ is an non-empty finite set of objects, A is a finite set of attributes and A is divided into a set C of conditional attributes and a set D of decision attributes.*

Definition 5.2. *Let $(\mathcal{U}, A)$ be an information system, let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and $X \subseteq \mathcal{U}$. Then the nano measure of X is defined as $\alpha(X) =$ 1- card $\left[\frac{\underline{R}(X)}{\overline{R}(X)}\right]$.*

Definition 5.3. *Let $(\mathcal{U}, A)$ be an information system,and $\mathcal{I}$ be an ideal on $\mathcal{U}$, $(\mathcal{U}, \tau_{R_{\mathcal{I}}(X)})$ be a R-nano ideal topological space and $X \subseteq \mathcal{U}$. Then the nano* *measure of X is defined as* $\alpha_{\mathcal{I}}(X)$ = 1- *card* $\left[\frac{\underline{R}_{\mathcal{I}}(X)}{\overline{R}_{\mathcal{I}(X)}}\right]$.

$\mathcal{U}$	Languages Spoken	Academic Qualification
P_1	$\{French, English\}$	$\{B.Sc., M.Sc., Ph.D\}$
P_2	$\{Hindi, Malayalam\}$	$\{B.Sc.,\}$
P_3	$\{French, English, Sanskrit\}$	$\{B.Sc., M.Sc.,\}$
P_4	$\{French\}$	$\{B.Sc., M.Sc.,\}$
P_5	$\{Malayalam\}$	$\{B.Sc.,\}$
P_6	$\{French, Sanskrit\}$	$\{B.Sc.,\}$

Table 2

Consider an example in Multi-valued information table of a file containing applicants $\mathcal{U} = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ for a job. It has conditional attributes of languages they speak and their scientific qualifications.

Relationships among the persons in set $\mathcal{U}$ are determined by the binary relations. These relations are referred to as information relations and they are determined by the problem.

We prefer persons who speak more scientific languages and have more scientific degrees, we choose the subset relations between the persons with respect to attributes: languages and scientific degrees.

So P_iRP_j iff $Lan[P_i \subseteq LanP_j]$ and $Qualfn.[P_i] \subseteq Qualfn.[P_j]$. Then $R = \{(P_1, P_1), (P_2, P_2), (P_3, P_3), (P_4, P_4), (P_5, P_5), (P_6, P_6), (P_4, P_1), (P_4, P_3), (P_5, P_2), (P_6, P_3)\}$ is reflexive.

Hence $< P_1R >= \{P_1\}$, $< P_2R >= \{P_2\}$, $< P_3R >= \{P_3\}$, $< P_4R >= \{P_1, P_3, P_4\}$, $< P_5R >= \{P_5, P_2\}$, $< P_6R >= \{P_6, P_3\}$.

Let $\mathcal{I} = \{\emptyset, \{P_1\}, \{P_5\}, \{P_1, P_5\}\}$ be an ideal on $\mathcal{U}$.

Let X $= \{P_2, P_4, P_6\} \subseteq \mathcal{U}$. Then the nano approximations induced by an ideal are $\underline{R}_{\mathcal{I}}(X) = \{P_1, P_2, P_5\}$, $\overline{R}_{\mathcal{I}}(X) = \mathcal{U}$ and $B_{\mathcal{I}}(X) = \{P_3, P_4, P_6\}$.

The nano measure $\alpha_{\mathcal{I}}$(X) = 1- *card* $\left[\frac{\underline{R}_{\mathcal{I}}(X)}{\overline{R}_{\mathcal{I}(X)}}\right] = \frac{1}{2}$.

On the other hand nano approximations based on reflexive relation are $\underline{R}(X) = \{P_2\}$, $\overline{R}(X) = \{P_2, P_4, P_5, P_6\}$, BND(X) $= \{P_4, P_5, P_6\}$ and the nano measure α(X) = 1- *card* $\left[\frac{\underline{R}(X)}{\overline{R}(X)}\right] = \frac{3}{4}$.

Hence we have $\underline{R}(X) \subseteq \underline{R}_{\mathcal{I}}(X) \subseteq X \subseteq \overline{R}_{\mathcal{I}}(X) \subseteq \overline{R}(X)$ and also $\alpha_{\mathcal{I}}$(X) $\leq \alpha$(X). Thus interms of nano accuracy, measure and approximations induced via ideal is much better than nano approximations.

6. Nano Ideal Topology Stimulated by Equivalence Relation

Here we define the approximations via ideal and try to compare them based on their measures and approximations with nano topology.

Definition 6.1. *Let $\mathcal{U}$ be an universe and $\mathcal{U}/R$ be any indiscerniblity relation on $\mathcal{U}$ and Let $X \subseteq \mathcal{U}$ and $\mathcal{I}$ be any ideal on $\mathcal{U}$. Then the lower and upper approximations of X are defined as follows:*

$$L_{R_{\mathcal{I}}}(X) = \bigcup\{x \in \mathcal{U} : R[X] \cap X^c \in \mathcal{I}\} \cap X,$$
$$U_{R_{\mathcal{I}}}(X) = \bigcup\{x \in \mathcal{U} : R[X] \cap X \notin \mathcal{I}\} \cup X,$$
$$B_{R_{\mathcal{I}}}(X) = U_{R_{\mathcal{I}}}(X) - L_{R_{\mathcal{I}}}(X).$$

Then $\tau_{R_{\mathcal{I}(X)}}) = \{\mathcal{U}, \emptyset, L_{R_{\mathcal{I}}}(X), U_{R_{\mathcal{I}}}(X), B_{R_{\mathcal{I}}}(X)\}$ is the nano topology induced by an ideal and hence $(\mathcal{U}, \tau_{R_{\mathcal{I}(X)}})$ is defined as the nano ideal topological space based on equivalence relation.

Example 6.2. Let $\mathcal{U} = \{a, b, c, d\}$ and $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d\}\}$ be any indiscerneblity relation defined on $\mathcal{U}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ be an ideal on $\mathcal{U}$ and let $X = \{c\} \subseteq \mathcal{U}$. Then $L_{R_{\mathcal{I}}}(X) = \{c\}$, $U_{R_{\mathcal{I}}}(X) = \{b, c\}$ and $B_{R_{\mathcal{I}}}(X) = \{b\}$ and nano topology induced by an ideal is given by $\tau_{R_{\mathcal{I}(X)}}) = \{\mathcal{U}, \emptyset, \{c\}, \{b, c\}, \{b\}\}$.

Proposition 6.3. *Let $(\mathcal{U}, R)$ be an approximation space and I be an ideal on $\mathcal{U}$ and $X, Y \subseteq \mathcal{U}$. Then*

(i) $L_{R\mathcal{I}}(X) \subseteq X \subseteq U_{R\mathcal{I}}(X)$
(ii) $L_{R\mathcal{I}}(\emptyset) = U_{R\mathcal{I}}(\emptyset)$
(iii) $L_{R\mathcal{I}}(\mathcal{U}) = U_{R\mathcal{I}}(\mathcal{U})$
(iv) *If $X \subseteq Y$ then $L_{R\mathcal{I}}(X) \subseteq L_{R\mathcal{I}}(Y)$ and $U_{R\mathcal{I}}(X) \subseteq U_{R\mathcal{I}}(Y)$*

7. Analysis Relying on Approximations

In this section we obtain both the nano topology and nano ideal topology based on equivalence relations and compare their approximations. Here again, the nano ideal topology leads to contraction of boundary.

Example 7.1. Let $\mathcal{U} = \{a, b, c, d\}$ be the universe and and $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d\}\}$ be any indiscerniblity relation defined on $\mathcal{U}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ be an ideal on $\mathcal{U}$. We find the approximations of all possible subsets of $\mathcal{U}$ in case of nano topology and nano ideal topology in the following table.

The above example of nano ideal topology based on equivalence relation leads us to a tool to minimise the boundary region to the least. Imprecision in this approach is expressed by boundary region of a set.

X	$L_{R_{\mathcal{I}}}(X)$	$L_R(X)$	$U_{R_{\mathcal{I}}}(X)$	$U_R(X)$	$B_{R_{\mathcal{I}}}(X)$	$B_R(X)$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\emptyset$	$\emptyset$
$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\emptyset$	$\emptyset$
$\{b\}$	$\{b\}$	$\emptyset$	$\{b,c\}$	$\{b,c\}$	$\{c\}$	$\{b,c\}$
$\{c\}$	$\emptyset$	$\emptyset$	$\{c\}$	$\{b,c\}$	$\{c\}$	$\{b,c\}$
$\{d\}$	$\{d\}$	$\{d\}$	$\{d\}$	$\{d\}$	$\emptyset$	$\emptyset$
$\{a,b\}$	$\{a,b\}$	$\{a\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{c\}$	$\{b,c\}$
$\{a,c\}$	$\{a\}$	$\{a\}$	$\{a,c\}$	$\{a,b,c\}$	$\{c\}$	$\{b,c\}$
$\{a,d\}$	$\{a,d\}$	$\{a\}$	$\{a,d\}$	$\{a,d\}$	$\emptyset$	$\{d\}$
$\{b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{b,c\}$	$\emptyset$	$\emptyset$
$\{b,d\}$	$\{b,d\}$	$\{d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{c\}$	$\{b,c\}$
$\{c,d\}$	$\{d\}$	$\{d\}$	$\{c,d\}$	$\{b,c,d\}$	$\{c\}$	$\{b,c\}$
$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\mathcal{U}$	$\emptyset$	$\{d\}$
$\{a,b,d\}$	$\{a,b,d\}$	$\{a,d\}$	$\mathcal{U}$	$\mathcal{U}$	$\{c\}$	$\{b,c\}$
$\{a,c,d\}$	$\{a,d\}$	$\{a,d\}$	$\{a,c,d\}$	$\mathcal{U}$	$\{c\}$	$\{b,c\}$
$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\emptyset$	$\emptyset$

Table 3

8. Illustration with Respect to Measures

There are a number of factors to consider when you start looking to buy a new car. If we look outside the budgetary aspects there are still a number of issues to consider. Here, we briefly describe car information table and involve that data as a topological comparison based on measures.

The following table contains information about Eight Cars characterised by four (attributes) which were used to decide the quality of each car (decision attribute), where the attributes are shown in the following table. The columns

Cars	Mileage	Interior	Noise	Vibration	Quality
C_1	Low	Fair	Medium	Medium	Low
C_2	Average	Good	Low	Low	High
C_3	Low	Good	Medium	Medium	Low
C_4	Average	Fair	Low	Low	Medium
C_5	High	Excellent	Low	Medium	High
C_6	Average	Fair	Medium	Low	Medium
C_7	High	Good	Low	Low	High
C_8	Low	Fair	Low	Medium	Low

Table 4

of the table represent the factors of a car and rows represent the Cars. The entries in the table are the attribute values. The given information system is complete and is given by $(\mathcal{U}, A)$ where $\mathcal{U} = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8\}$ and $A = \{Mileage, Interior, Noise, Vibration\}$ which is divided into a set C of condition attributes and a set Quality of car as decision attribute. Based on the decision attribute $\mathcal{U}/R(D) = \{\{C_1, C_3, C_8\}, \{C_4, C_6\}, \{, C_2 C_5, C_7\}$. Let I be an Ideal on $\mathcal{U}$ where $\mathcal{I} = \{\{C_8, C_3, C_4\}, \{C_8, C_3\}, \{C_8, C_4\}, \{C_3, C_4\}, \{C_8\}, \{C_3\}, \{C_4\}, \emptyset\}$.

Let $X = \{\{C_4, C_5, C_6, C_8\}\} \subseteq \mathcal{U}$, $X^c = \{C_1, C_2, C_3, C_7\}$. Then $L_{R_\mathcal{I}}(X) = \{C_4, C_5, C_6, C_8\}$, $U_{R_\mathcal{I}}(X) = \{C_2, C_4, C_5, C_6, C_7, C_8\}$ and $B_{R_\mathcal{I}}(X) = \{C_2, C_7\}$. Hence the nano topology induced by an ideal is given by $\tau_{R\mathcal{I}(X)} = \{\mathcal{U}, \emptyset, \{C_4, C_5, C_6, C_8\}, \{C_2, C_4, C_5, C_6, C_7, C_8\}, \{C_2, C_7\}\}$.

The nano accuracy $\alpha_\mathcal{I}$(X) = 1- $card\left[\frac{L_{R\mathcal{I}}(X)}{U_{R\mathcal{I}(X)}}\right] = \frac{1}{2}$.

On the other hand according to nano approximations are $L_R(X) = \{C_4, C_6\}$, $U_R(X) = \mathcal{U}$, $B_R(X) = \{C_1, C_2, C_3, C_5, C_7, C_8\}$ and the nano accuracy α(X) = 1-$card\left[\frac{\underline{\underline{R}}(X)}{\overline{R}(X)}\right] = \frac{3}{4}$.

Hence we have $L_R(X) \subseteq L_{R\mathcal{I}}(X) \subseteq X \subseteq U_{R\mathcal{I}}(X) \subseteq U_R(X)$ and also $\alpha_\mathcal{I}$(X) $\leq \alpha$(X).

Hence we can conclude by means of accuracy measure and in terms of approximations nano topology via ideal is superior to the existing one.

Conclusion 8.1. In this paper, new definitions of lower and upper approximations via ideal have been introduced for reflexive binary relations and equivalence relations. These new definitions are compared with nano approximations. It's therefore shown that the current definitions have additional generalization. It is revealed that the Nano ideal topology decreases the boundary region and we get a topology finer than nano topology. Nano ideal topology based on equivalence relation is useful in the analysis of data presented in terms of complete information systems. Due to imprecise human knowledge, sometimes it is not possible to find an equivalence relation among the elements of the universe set $\mathcal{U}$. Therefore we sensed the need for nano ideal topology based on reflexive relations more general than equivalence relations. Further this method is accentuated in the cases of Multi-valued information systems and incomplete information system. We believe that the approaches we have offered here will turn out to be more useful for practical applications of the nano topological theory and help us to gain much more insights into the mathematical structures of nano approximation operators.

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