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Memories

A short biography of Professor Ky Fan (樊士畿), one of the greatest mathematicians of the 20th century

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It is a great honor for me to present here an outline of the life, and great achievements and contributions of Professor Ky Fan, and also recall my interactions with him.

I heard of Ky Fan's name for the first time in 1987 from a colleague in Bonn, Germany, who had met him at a conference in Heidelberg. I was told that Ky Fan was famous for his Ky Fan lemmas and inequalities, and he had an air of authority, speaking in a straightforward manner. I was very impressed. Later I somehow got the wrong impression that S.T. Yau, together with W. Meeks III, applied a lemma of Ky Fan in a paper on minimal surfaces in 3-manifolds. In the process of writing this article I looked through the papers by Yau on minimal surfaces and didn't find any example of applying a lemma of Ky Fan. Of course, this has no implication about the significance of Ky Fan's lemmas. Since I worked on minimal surfaces (for example, my Ph.D. thesis, written under the direction of Prof. S. Hildebrandt, is about minimal surfaces with free boundary), this was particularly interesting to me. When I arrived in Santa Barbara in 1990, Ky Fan had already retired. But he still came to his office in the Mathematics Department at the University of California, Santa Barbara almost daily. I was the only Chinese faculty in the department at that time, so it was natural for him to be interested in chatting with me, although my area of research had no intersection with his. (Somehow I never asked him about my wrong impression mentioned above). It was quite a few years later when I came to realize how great a mathematician Ky Fan was. Indeed, I learned that Ky Fan was as famous as S.S. Chern, one of the greatest mathematicians and the greatest geometer in the 20th century. Often one also groups Ky Fan together with Chern and another great Chinese mathematician Luo-keng Hua. I was very fortunate, along with two other Chinese Ph.D. students, to receive the generous and invaluable attention and caring from Professor Chern when he visited the University of Bonn in Germany on quite a few occasions. On top of that, I was then so fortunate to come to UC Santa Barbara and thereupon also had interactions and a friendship with Ky Fan over many years.

I had many conversations with Ky Fan alone, most of which were short. Among the longer conversations I remember most vividly one in the beautiful beachside restaurant close to the campus, where I had dinner with him as my guest. We talked about mathematics, and the history and contemporary developments of China, among other topics.

Ky Fan passed away on March 22, 2010 at the age of 95 in Santa Barbara. I organized and hosted the funeral ceremony of Ky Fan for his family and the family friends, the Mathematics Department at UC Santa Barbara, and UC Santa Barbara. I'd like to include here a short poem I wrote then for the remembrance of Ky Fan. The poem is phrased to sound like it was written by Ky Fan himself. I hope that this poem does convey his thoughts and emotions to some extent at least.

The Chinese version:

叹数理之深奥兮吾层层探究终身不息
感宇宙之浩淼兮吾上下翱翔其乐无穷
念伴侣之至爱兮吾执手凝望恋恋不舍
期人类之昌盛兮吾诚愿再尽绵薄之力

The English version:

Alas, mathematics is so deep and complex,

In which I have researched in all ways all my life.

Alas, the universe is so vast,

In which I have flown hither and yon with great joy.

The deep love of my companions has been so dear to me;

I held their hands and couldn't let them go.

I want mankind to prosper, and I am so eager

To make my humble contribution once again.

A couple of months ago I visited Ky Fan's house and talked to his second wife Xiaoxia Wang again (she taught mathematics at Renming University in Beijing many years ago). A winding uphill road leads to the house which is located on a hilltop. The front of the house faces the mountains and in particular their peaks, while the back of the house overlooks the ocean in some distance. The backyard and side yard provide spectacular panoramic views of the ocean, the city, the flat land, the valleys, and the mountains.

The beautiful Santa Barbara was an ideal place for Ky Fan to ponder many research problems of mathematics, and UC Santa Barbara was an ideal campus for him to teach mathematics and train his Ph.D. students.

Ky Fan's Early Years

Ky Fan was born on September 19, 1914, in the beautiful city Hangzhou, China. This city and another city Suzhou are said in China to rival Heaven in their beauty and prosperity. His father Qi Fan served at the district courts of the cities Jinhua and Wenzhou. Ky Fan studied at several schools in Hangzhou and other cities and earned top grades in all courses. He enrolled in Peking University, i.e. Beijing University, in 1932. Originally, he intended to study engineering. Under the influence of his uncle Zuxun Feng, the chair of the Mathematics Department at Peking University, he chose to study mathematics instead. One can say that he was destined to play a major role later in the developments of several important fields of mathematics.

The young Ky Fan excelled in mathematics with ease. During the summer of his second year at the university he translated the German textbooks *Einführung in die*

Analytische Geometrie und Algebra (by O. Schreier and E. Sperner) and *Vorlesungen über Matrizen* into Chinese and combined them into a two-volume textbook *Analytic Geometry and Algebra I, II*, which was then adopted as a standard textbook in China for many years. During his four years of study at Peking University, Ky Fan also produced a translation of Landau's textbook on algebraic number and ideal theory, and wrote a book on number theory together with a fellow student.

After receiving his B.S. degree from Peking University in 1936, Ky Fan was hired by the university as an instructor. In 1939, he was selected by the China-France Education Foundation and went to Paris to pursue graduate study in mathematics. Two years later, i.e. in 1941, Ky Fan completed his thesis "Sur quelques notions fondamentales de l'analyse générale" under the direction of Maurice Fréchet and received his Doctorate of Science from University of Paris. Then he became a French National Science Fellow at Centre National de la Recherche Scientifique and a member of Institut Henri Poincaré later. In 1945, at the end of WWII, Ky Fan had already published 25 research papers on his own and the monograph *Introduction a la Topologie Combinatoire, I. Initiation*, which he co-wrote with Fréchet.

Ky Fan in the Unites States

Ky Fan came to the Unites States in 1945. He was a member of the Institute for Advanced Study in Princeton from 1945 to 1947, when Hermann Weyl and John von Neumann were also there. Then he joined the mathematics faculty of the University of Notre Dame and taught there from 1947 to 1960, first as an assistant professor, and later as an associate professor and a full professor. During this period, Ky Fan conducted summer research at the National Bureau of Standards, Oak Ridge National Laboratory, and Argonne National Laboratory. He taught at Wayne State University from 1960 to 1961 and at Northwestern University from 1961 to 1965.

In 1965 Ky Fan settled in Santa Barbara. He taught at UC Santa Barbara from 1965 until his retirement in 1985. He was chair of the Mathematics Department at UC Santa Barbara from 1968 to 1969.

Upon Ky Fan's retirement, the Ky Fan Visiting Assistant Professorship in the Mathematics Department at UC Santa Barbara was founded with his donation. VAPs (visiting assistant professors) are very much needed for our teaching programs, and they also very energetically participate in our research activities. The Ky Fan Visiting Professorship provides a lasting VAP position and has played an important role ever since its inception.

Ky Fan's Mathematical Achievements and Influences

A.

Ky Fan was one of the greatest mathematicians of the twentieth century. His mathematical career spanned more than seventy years, and his mathematical achievements were exceptionally versatile. He made fundamental and groundbreaking contributions in linear and nonlinear analysis, convex analysis and inequalities, fixed point theory, operator and matrix theory, linear and nonlinear programming, complex analysis, topology, and topological groups. His influence and impact in those fields and other fields are profound and lasting.

Ky Fan's mathematical research is often concerned with the foundation and central issues of a field or direction, and extends from finite dimensions to infinite dimensions and from theoretic to applied mathematics. He pioneered a number of new research directions and many of his results have become classics. A large number of important mathematical results bear Ky Fan's name and are constantly cited in the literature of many fields of mathematics, such as Ky Fan theorems, Ky Fan lemma, Ky Fan conditions, Ky Fan maximum principle, Ky Fan minimax equalities and inequalities, and Ky Fan norms, etc. Ky Fan's mathematical works have found wide applications in many fields. For example, his work in fixed point theory, in addition to influencing nonlinear functional analysis, has found wide applications in mathematical economics and game theory, potential theory, calculus of variations, and differential equations.

Ky Fan's 140 papers and books have been cited over 4,000 times.

As mentioned before, my areas of research have overall been rather remote from Ky Fan's. However, I have been very interested in a number of areas of analysis, some of which are needed for geometric analysis, one of my research fields. As an example let me mention the role played by convex analysis in Richard Hamilton's theory of the Ricci flow. Basically, one shows that a given suitable convex condition about curvature is preserved by the so-called Hamilton ODE. Then it follows from a maximum principle argument that the condition is preserved along the Ricci flow. Ky Fan's important work in convex analysis seems to be of a very different nature, but it is worthwhile to take a broader view of convex analysis and try to find some connections here.

Another example is matrix theory. The father of a college student studying a difficult homework problem about matrices asked me for help, as I was his friend. I did some initial calculation and realized that I didn't have enough time to solve the problem (or I didn't find some smart idea which could be used to crack the nut), so I searched for a solution in the literature. Then I found a paper by Ky Fan which thoroughly treated a more general problem. Ky Fan's depth of thinking and thoroughness therein were striking.

B.

Ky Fan had a number of honorary doctorates and professorships from famous universities such as Beijing University, Beijing Normal University, and Université de Paris-Dauphine. He held a number of visiting positions at e.g. University of Texas-Austin, Universität Hamburg, Université Paris IX, and Università degli Studi di Perugia. In 1964, Ky Fan was elected a member of Academia Sinica in Taiwan, China. From 1978 to 1984, he also served twice as the director of the Mathematics Institute of Academia Sinica in Taiwan, China.

Ky Fan served on many editorial boards. He was honored with the title "Distinguished Editor of Linear Algebra and Its Applications". He was one of the founding editors and served as the chief editor of the Journal of Mathematical Analysis and Applications and the Journal of Nonlinear and Convex Analysis.

In 1984, a large international mathematical conference was held at UC Santa Barbara to celebrate Ky Fan's contributions to mathematics on the occasion of his upcoming retirement in 1985. The book *Nonlinear and Convex Analysis: Proceedings in Honor of Ky Fan*, comprised of papers presented at the conference, was then published. An entire issue of the journal *Topological Methods in Nonlinear Analysis* was dedicated to Ky Fan on the occasion of his 80th birthday in 1994. Two issues of the *Bulletin of the Institute of Mathematics, Academia Sinica* — Vol.2, No.2, 1974, and Vol.3, No.1, 1975 — were dedicated to him for his sixtieth birthday. In 1993 the T.I.Tec/K.E.S. Conference on Nonlinear and Convex Analysis in Tokyo was dedicated to Ky Fan in recognition of his fundamental contributions to the field. In 2011, the seventh International Conference on Nonlinear and Convex Analysis in Hirosaki was dedicated to his memory.

C.

Ky Fan was a great teacher. His courses were unusually rich in content. The content of each of his (graduate) courses was usually about double the standard requirement. His lectures were always perfectly organized and elegantly presented. He prepared new lecture notes each time he taught a course. Ky Fan was known to be a very strict and demanding teacher. For example, he assigned very difficult problems for homework and in exams of graduate courses. He wanted total devotion from all his students. He constantly pushed the students to study, to work, and to think. Once Ky Fan told the graduate students in the Mathematics Department of UC Santa Barbara to “think on mathematics at very waking moment”. Clearly, he was an exemplary practitioner of this famous motto of him.

On the other hand, Ky Fan was always kind and constantly helped his students and many other people in mathematics. He cared much about his students and often talked about their new research results.

The last course Ky Fan taught was “Topological Groups” for senior graduate students of the Mathematics Department at UC Santa Barbara. Ky Fan had 21 Ph.D. students and 89 mathematical descendants.

Ky Fan and China

As the director of the Mathematics Institute of Academia Sinica in Taiwan, China for two terms, Ky Fan had a vision for mathematical research in Taiwan and played a pivotal role for the development of the Institute. In particular, he made great efforts to recruit young mathematicians (he did the same at UC Santa Barbara and I'm sure also at other universities where he taught).

After 50 years of absence, Ky Fan returned to his motherland Mainland China in May, 1989. His visit — the visit of such a world-renowned mathematician — was a big event for the mathematical community of China. During the visit he received the honorary professorships from Beijing University, his Alma Mater, and Beijing Normal University which were mentioned before. In the subsequent years Ky Fan visited China a couple more times. Ky Fan cared much about the development of mathematics in China. He did a great deal to help young Chinese mathematicians and made great contributions to exchanges and collaborations between the mathematical communities of China and the United States. In this regard I must mention the important “Ky and Yu-Fen Fan Endowment” for supporting Chinese mathematics and promoting cooperation between mathematicians in China and North America. It was in 1999 that Ky Fan and his first wife Yu-Fen Fan donated one million US dollars to the American Mathematical Society to create this Endowment. I would also like to mention that a number of Chinese mathematicians and graduate students came to UC Santa Barbara to be visiting scholars sponsored or Ph.D. students supervised by Ky Fan, and that Ky Fan donated almost all of his collection of mathematical books and magazines to Beijing University in 1987.

In conclusion I would like to say that Ky Fan's mathematical heritage is extraordinarily rich, and his brilliance in mathematics and great personality will be forever remembered.

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